

Chapter 6: Applications of Integrals

Review Assignment

Name:

Date:

Your Target Grade For This Chapter: (MUST Indicate)
Please CIRCLE One Of: A+ A B C
(This helps the marker give you feedback.)

- This assignment is to help YOU. It will identify 3 things for you:

- How much of this chapter you really understand,
- What you need more practice on before the test,
- And whether you're doing each step correctly.

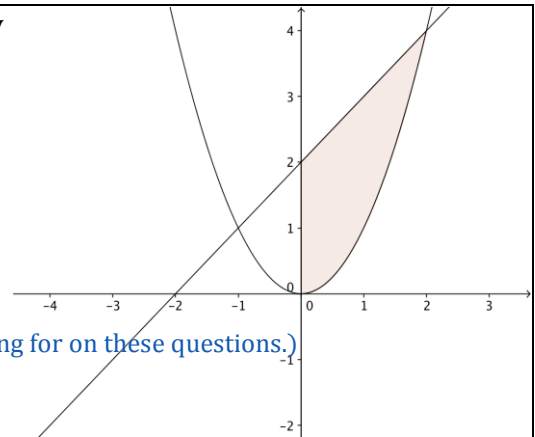
- Some of the answers are on the last page – what we want to see

is your work. So show all necessary work, or you'll see stuff written down *right around here...* ↑ 😊

This assignment...

Is Good To Go!	Needs More Work On Question(s):

1. Consider the region in the first quadrant that is enclosed by the functions $y = x^2$, $y = x + 2$, and $x = 0$. For each of the following, set up the definite integral(s) that would solve the question, but don't simplify it at all, and don't evaluate it either! (but know how to do that, of course, for the test)



a) Find the area of the region by integrating with respect to x.

Answer: *Area = top function - bottom function*

$$A = \int_0^2 [(x+2) - (x^2)] dx$$

(Note: Answer given for a) to show you how much work we're looking for on these questions.)

b) Find the area of the region by integrating with respect to y.

$$x = \sqrt{y}, \quad x = y - 2$$

$$A = \int_0^2 \sqrt{y} dy + \int_2^4 [\sqrt{y} - (y-2)] dy = \int_0^2 \sqrt{y} dy + \int_2^4 (\sqrt{y} - y + 2) dy$$

c) The region is rotated around the x axis - find the volume by the method of discs.

$$x^2 = x + 2 \quad (x-2)(x+1) = 0, \quad x = -1, 2, \text{ given } (0, 2)$$

$$V = \int_0^2 \pi [(x+2)^2 - (x^2)^2] dx = \pi \int_0^2 (-x^4 + x^2 + 2x + 4) dx$$

d) The region is rotated around the y axis - find the volume by the method of shells.

$$dV = 2\pi(x)(x+2-x^2) dx$$

$$V = \int dV = \int_0^2 2\pi(x^2+2x-x^3) dx = 2\pi \int (x^2+2x-x^3) dx$$

e) The region is rotated around the line $y = -2$. Find its volume using discs.

$$dV = \pi(R^2 - r^2) \cdot h \rightarrow dV = \pi[(y_1+2)^2 - (y_2+2)^2] dx$$

$$V = \int dV = \int_0^2 \pi[(x+2+2)^2 - (x^2+2)^2] dx$$

$$= \pi \int_0^2 (-x^4 - 3x^2 + 8x + 12) dx$$

f) The region is rotated around the line $x = -4$. Find its volume using shells.

$$dV = 2\pi r h (\text{thickness}), \quad r = x+4, \quad h = (x+2) - x^2$$

$$V = \int dV = \int_0^2 2\pi(x+4)(x+2-x^2) dx$$

$$= 2\pi \int_0^2 (-x^3 - 3x^2 + 6x + 8) dx$$

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2. When a 1.0 N force is applied to a spring, it is stretched 0.25 m from its resting position. Calculate the work done on the spring to stretch it from its resting position to that distance.

$$x = 0.25 \text{ m}, F(x) = 1.0 \text{ N}$$

$$F = kx \quad 1 = 0.25k$$

$$k = 4, F = 4x$$

$$W = \int_0^{0.25} (4x) dx$$

$$= \left[4x \frac{x^2}{2} \right]_0^{0.25}$$

$$= 2(0.25)^2 = 0.125 \text{ (J)}$$

3. What is the average value of $f(x) = e^x - e^{-x}$ on the interval $[1, 3]$? (Round to 2 dec. places.)

4. A thick 10 m rope that weighs a total of 200 N is hanging over the edge of a cliff. How much work does it take to lift the entire rope up? (Try this, and if you get stuck, see the HINT in the Answers below.)

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5. A car which begins at position $d = 0$, travels according to the following velocity function: $v = 10t - 4t^2 m/s$. At the same time, at a position of $d = 5$ m, a truck begins at rest, and travels at a constant acceleration, $a = 6 m/s$.

a) Find the acceleration function and the position function of the car.

b) Find the velocity function and the position function of the truck.

c) What's the car's displacement in the first 3 seconds? (**solve algebraically**)

d) What's the distance the car travels in the first 3 seconds? (**show the integral set up first, and then use the graph. calc. to solve**)

e) How far apart are the two vehicles after 3 sec? (**solve algebraically**)

6. **Using full sentence(s)**, explain why the definite integral of a function that is below the x-axis on a certain interval will result in a negative value. Hint: If you want, you can try watching Chapter **5** Lesson 2 Page 2 again.)

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(No need to upload this page)

CHECK WITH THESE ANSWERS *BEFORE* SUBMITTING

(Remember: Do these on your own, and submit your best effort.)

1.

a) $A = \int_0^2 [(x + 2) - (x^2)] dx$

b) ----- (Hint: Break the region into two parts by drawing a dotted line at $y = 2$.)

c) -----

d) -----

e) -----

f) -----

2. 0.125 J

3. ≈ 8.52

4. 1000 J

Hint: Calculate the little bit of work, dW , that it takes to lift a little piece of rope, dx , a distance of "x" to the top of the cliff. And note that the little bit of force, dF , on a little piece of rope, dx , is:

$dF = 20 dx$. And that's because $dF = \frac{100(N)}{20(m)} dx (m) = 20 dx (N)$.

5.

a) $a(t) = 10 - 8t$ $d(t) = 5t^2 - \frac{4}{3}t^3$

b) $v(t) = 6t$ $d(t) = 3t^2 + 5$

c) 9 m

d) ≈ 11.83 m

e) 23 m

6. -----