Ch 7 Practice Test – Differential Equations and Numerical Methods

1. Solve the differential equation: $y^3y' = x^4 + x$.

2. For the differential equation: $\cos^2 x \frac{dy}{dx} = \frac{y}{\ln y}$,
   a) Give the general solution.
   b) Find the unique solution that satisfies $y = e^{-2}$ when x =0.

3. The velocity of a skydiver, v(t), in feet per second, satisfies the differential equation $\frac{dv}{dt} = -3v - 32$, and v(0)=0.
a) Find v in terms of t.
b) The terminal velocity is given by $\lim_{t\to\infty} v(t)$. Find the terminal velocity.

4. Suppose that a certain animal population is decreasing at a rate of 3% per year, and that y is the number of bacteria present after t hours.
a) Write the differential decay model that represents this situation.
b) If the initial population is 25,000, write an expression for the population, y, as a function of the time, t, in years.

5. If the half-life of a substance is 1.3 days, how long until this substance is reduced to 1% of its initial amount?

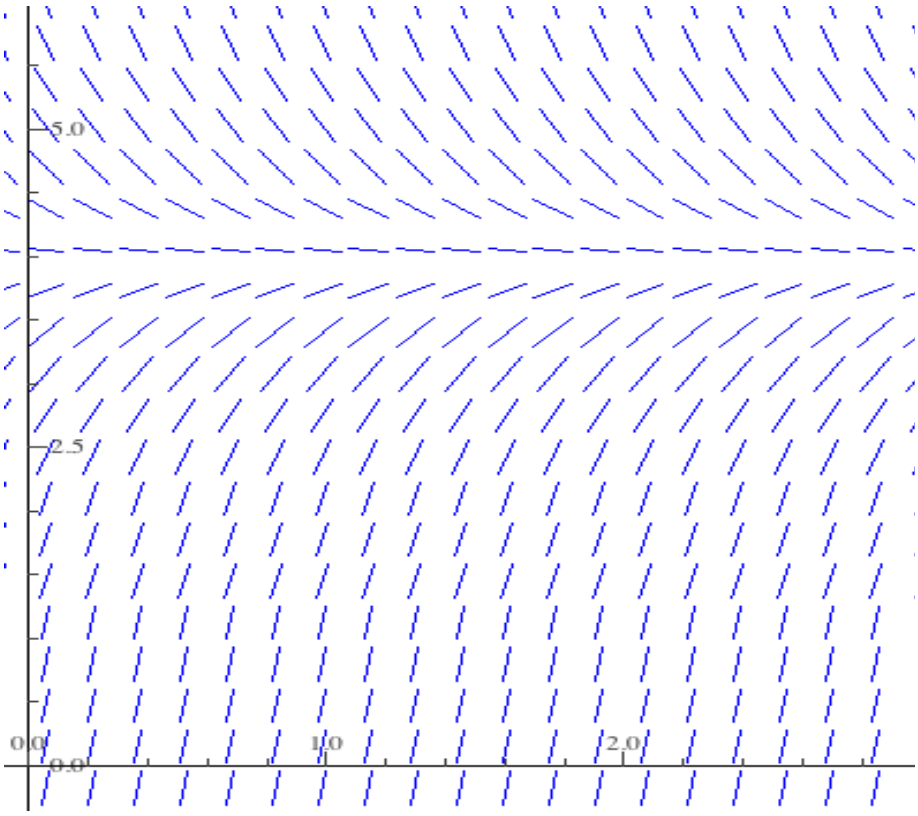
6. From 2001 to 2011, the rate of internet use worldwide has grown by nearly 500%. Assuming this growth rate of 500% for every 10 years continues,
a) Write the differential growth model that represents this situation.
b) Given a current population of internet users of 2.3 billion, how long will it take until 5 billion people are using the internet?

7. Answer true or false. The differential equation $y'' + 2y = -3y'$ is solved by $y = Ce^{3t}$.

8. What is the general solution of the differential equation $dy/dt - 9\cot y = 0$?

9. If $y' = x\cos y$, then what is the slope of the direction field at the point (5, 0)?

10. The current flow for a basic electrical system is described by the following differential equation: $\frac{dI}{dt} = 12 - 3I$, where I is the current (in Amps) and t is the time (in secs). Given its slope field below, sketch the solution curve that, at time = 1 sec, has a current of zero (0).



11. For the differential equation $3\frac{dy}{dx} - 15y^2 = \cos x$, $y(0.3) = 5$,
and using a step size of h=0.3, what is the value of $y(0.9)$ to 4 dec. places, using Euler's method?

12. a) Find the linear approximation, L(x), of the function $f(x) = \sqrt[3]{2+x}$ at the point $x = 0$, and use it to approximate $\sqrt[3]{2.2}$.

b) What is the percent error of this approximation?

13. When solving the equation $3x - \cos x = 0$ using Newton's method with initial approximation $x = 0$, what is x_3 to 4 dec. places?

14. To approximate the integral $\int_{0.6}^{2.4} \ln x \ dx$, suppose the interval is divided into equal parts each of width 0.3. What is the approximation for the value of the integral using:

a) The Trapezoidal Rule

b) Simpson's Rule