Calculus

Chapter 5: Integration

Lesson 1: Area Approximation and Riemann Sums

Question #1

Reference Q.378

Estimate the area under the following curve using the suggested Rectangular Approximation Method.

$f(x) = \sqrt[3]{x}$ on the interval [0,2] (Use the a) Left End Point and b) Right End Point for the height in each interval, and 4 rectangles)

a. Left End Point

b. Right End Point

Question #2

Reference Q.379

a. Set up and evaluate the Riemann Sum for $f(x) = \cos x$ on the interval $[0, \pi]$ using 3 intervals, and use the midpoint to determine the height in each interval.

b. Why isn't the result from a) some larger, positive value?

Question #3

Reference Q.380

Estimate the area under the following curve using the suggested Rectangular Approximation Method.

$f(x) = \frac{1}{2x^2}$ on the interval [-3, -1] (Use the (a) Left, (b) Right, and (c) Middle endpoint for the height in each interval, and 5 rectangles)

a. Left

b. Right

c. Middle endpoint

Question #4

Reference Q.381

Evaluate the following to practice the basic skills of Sigma Notation.

$$\sum_{i=1}^{3} i^4$$

Question #5

Reference Q.382

Evaluate the following to practice the basic skills of Sigma Notation.

$$\sum_{k=-4}^{1} \left(2k^2 - 2k + 1\right)$$

Question #6

Reference Q.383

Write in sigma notation (but don't evaluate):

$3 + 6 + 9 + 12 + \ldots + 21$

Question #7

Reference Q.384

Divide $f(x) = 2x + 2$ on the interval $[-1, 3]$ into 4 subintervals of equal size and then compute $\sum_{k=1}^{4} f(x_k^*)\Delta x$ with $x*_k$ as the left point of each subinterval

Question #8

Reference Q.385

Divide $f(x) = 2x + 2$ on the interval $[-1, 3]$ into 4 subintervals of equal size and then compute $\sum_{k=1}^{4} f(x_k^*)\Delta x$ with $x*_k$ as the right endpoint of each subinterval.

Question #9

Reference Q.386

Divide $f(x) = 2x + 2$ on the interval $[-1, 3]$ into 4 subintervals of equal size and then compute $\sum_{k=1}^{4} f(x_k^*)\Delta x$ with $x*_k$ as the midpoint of each subinterval.

Question #10

Reference Q.387

If you have a special program in your graphing calculator to do this (yes, you can find those! You can also program it yourself using the PRGM function...), find the area under the curve $f(x) = \frac{1}{x^2}$ on [1,2] using left end, right end, and midpoint, approximations for 10, 20 and 50 divisions.

Question #11

Reference Q.388

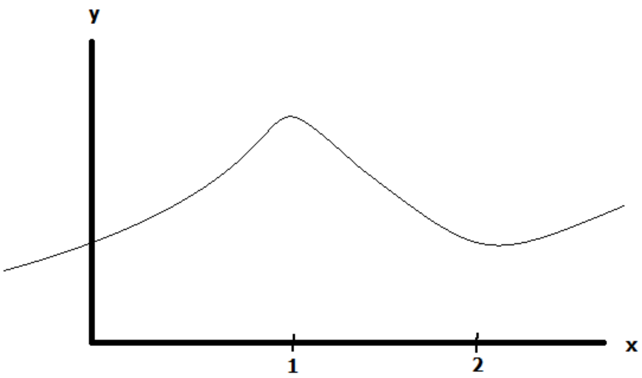
Find $\sum_{i=a}^{b} f(x*_i)\triangle x$

$f(x) = x - 1;\ a = 0;\ b = 4;\ n = 4;\ \triangle x_1 = 2;\ \triangle x_2 = 1;\ \triangle x_3 = 0.7;\ \triangle x_4 = 0.3;\ x*_1 = 1.5;\ x*_2 = 1.1;\ x_3^* = 2.5;\ x_4^* = 2.8$

Question #13

Reference Q.9219

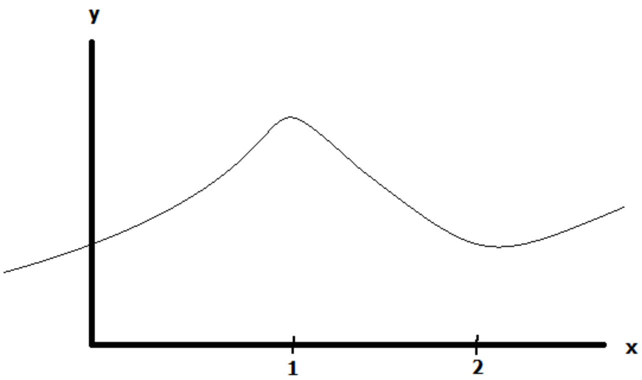
A right and left Riemann sum are used to approximate the area under the function depicted below between $x = 0$ and $x = 1$. Which sum gives an overestimate?



Question #14

Reference Q.9220

A right and left Riemann sum are used to approximate the area under the function depicted below between $x = 0$ and $x = 1$. Which sum gives an underestimate?



Question #12

Reference Q.389

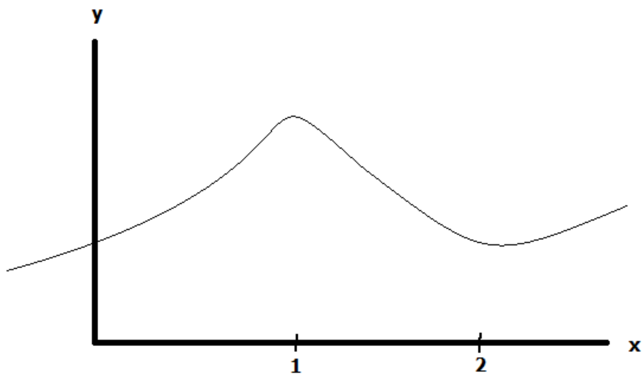
Find $\sum_{i=a}^{b} f(x *_i)\triangle x$

$f(x) = x^2 - 1;\ a = -3;\ b = 5;\ n = 3;\ \triangle x_1 = 2;\ \triangle x_2 = 1;\ \triangle x_3 = 5;\ x_1^* = -1.5;\ x_2^* = -0.1;\ x_3^* = 3.5$

Question #15

Reference Q.9221

A right and left Riemann sum are used to approximate the area under the function depicted below between $x = 1$ and $x = 2$. Which sum gives an overestimate?



Question #16

Reference Q.9222

A right and left Riemann sum are used to approximate the area under the function depicted below between $x = 1$ and $x = 2$. Which sum gives an underestimate?