Calculus

Chapter 6: Applications of Integrals

Lesson 1: Finding The Area Under and Between Curves

Question #1

Reference Q.592

Find the area under the curve $y = x^3 + x$ from x = 0 to x = 4.

Question #2

Reference Q.593

Find the area under the curve $y = \sqrt{2x} + x$ from x = 1 to x = 9.

Question #3

Reference Q.594

Find the area under the curve $y = \sin^2 x \cos x$ from x = -π/2 to x = π/2

Question #4

Reference Q.595

Find the area between the curves $y = \sqrt{x} + 1$ and $y = x - 1$ from 0 to 4.

Question #5

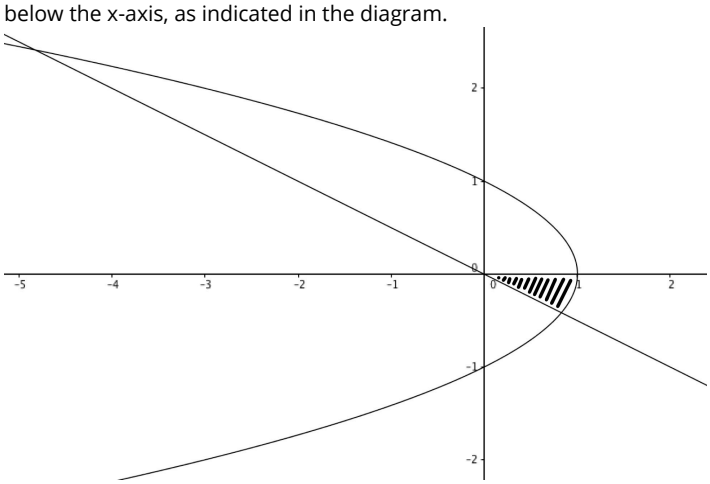
Reference Q.597

Find the area between the curves $y = x^2 + 2$ and $y = x - 1$ from -1 to +1.

Question #6

Reference Q.598

Find the area between the curves $x = 1 - y^2$ and $x = -2y$ and below the x-axis, as indicated in the diagram.



Question #7

Reference Q.599

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

$x = \cos y \ , x = 0, \ from\ y = 0\ to\ \frac{\pi}{4}$

a. sketch

b. area

Question #8

Reference Q.600

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

$x = 5 - y^2, \ x = y - 1$

a. sketch

b. area

Question #9

Reference Q.601

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

y= $2x^2,\ y = 2\sqrt{x},\ from\ x = 0\ to\ 1$

a. sketch

b. area

Question #10

Reference Q.602

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

$y = x^2 + 2,\ y = 5$

a. sketch

b. area

Question #11

Reference Q.603

Find the upper area between the curves $x - 4y^2 = 0$ and $x - 4y = 0$ in two different ways: by integrating with respect to x and then by integrating with respect to y. (They should yield the same answer!)

Question #12

Reference Q.604

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

$y = 8\sin x,\ y = \csc^2 x,$ from $x = \frac{\pi}{6}$ to $x = \frac{5\pi}{6}$

Question #13

Reference Q.605

For the following question, provide a sketch of the area between the two curves, and then calculate the actual area.

$y = x,\ y = 3x,\ y = 4 - x$

Question #14

Reference Q.606

Using a graphing calculator, determine the area of each of the enclosed regions below.

a. a) $y = 2\sin x\ , y = 2\cos x\ \text{from}\ x = \frac{\pi}{4}\ \text{to}\ \frac{5\pi}{4}$

b. b) $x = 3y^3 - 3y,\ x = 0$

Question #15

Reference Q.9242

A region in the xy-plane is bounded by the graph of $y = \frac{2}{x}$, the x-axis, the line $x = a$, and the line $x = 3a$, where a is a positive real number. Which of the following statements about the area of this region are true?

(A) It increases proportionately with a.
(B) It decreases when a increases.
(C) Its magnitude is independent of a.
(D) It would be undefined were a negative.

Question #16

Reference Q.9243

The area under $f(x) = \sin x$ between $\frac{-\pi}{4}$ and $\frac{3\pi}{4}$ has been sliced in half by a vertical line $x = c$. If the area between $\frac{-\pi}{4}$ and c is half as much as the area between c and $\frac{3\pi}{4}$, what is c?

Question #17

Reference Q.9244

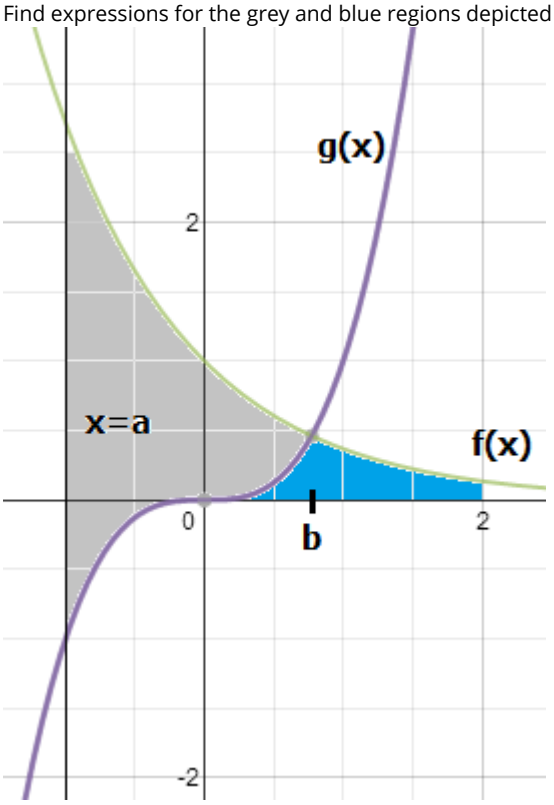
What is the area enclosed between $f(x) = \frac{2}{x}$ and $g(x) = 3 - x$?

Sketch the region.

Question #18

Reference Q.9245

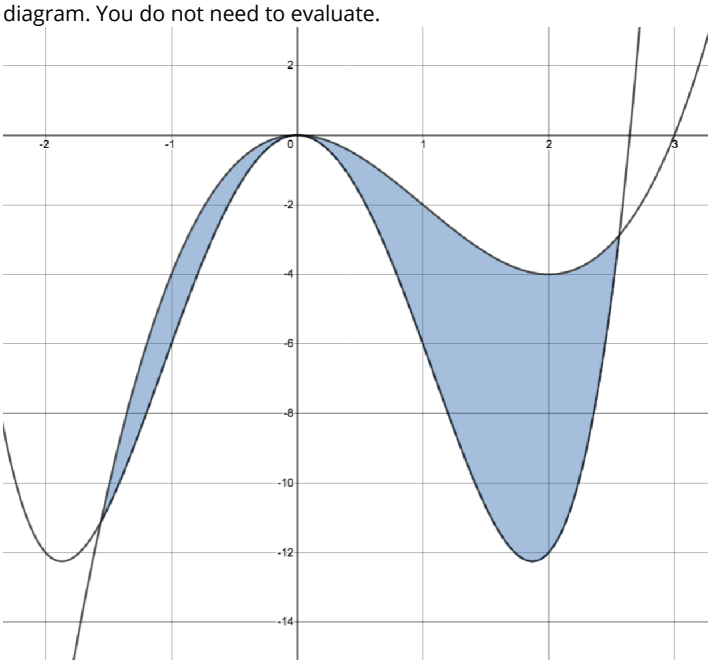
Find expressions for the grey and blue regions depicted below.



Question #19

Reference Q.50194

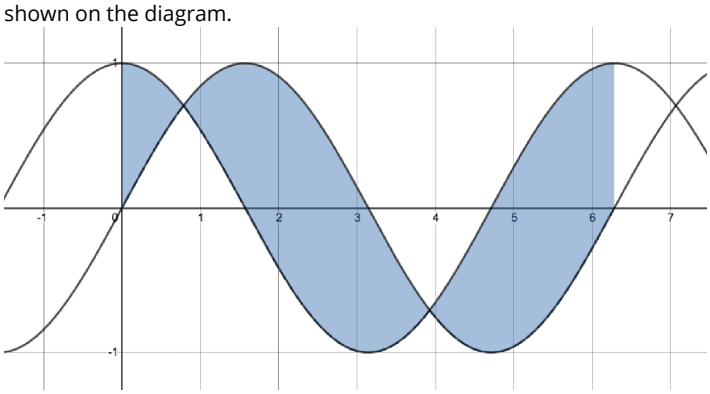
Set up an integral, or series of integrals, to calculate the area between the curves $y = x^4 - 7x^2$ and $y = x^3 - 3x^2$ as shown in the diagram. You do not need to evaluate.



Question #20

Reference Q.50197

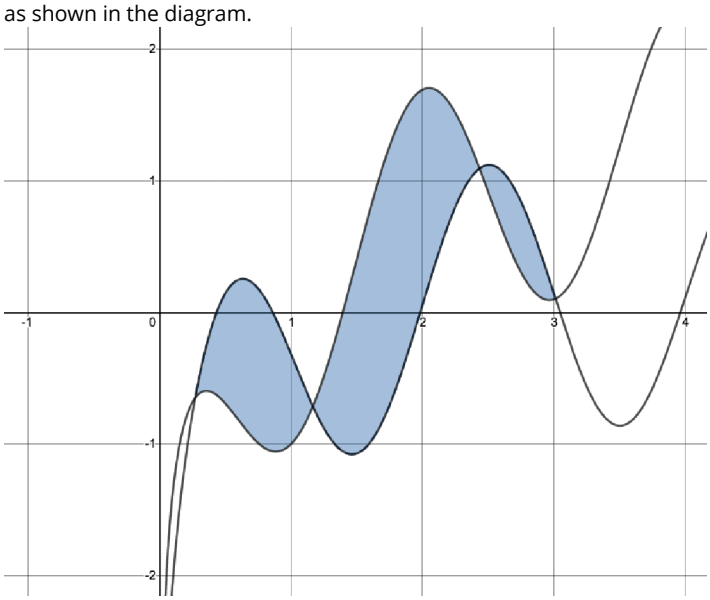
Using algebraic techniques, calculate the shaded area between the curves $y = sinx$ and $y = cosx$ on the domain $0 \leq x \leq 2\pi$, as shown on the diagram.



Question #21

Reference Q.50200

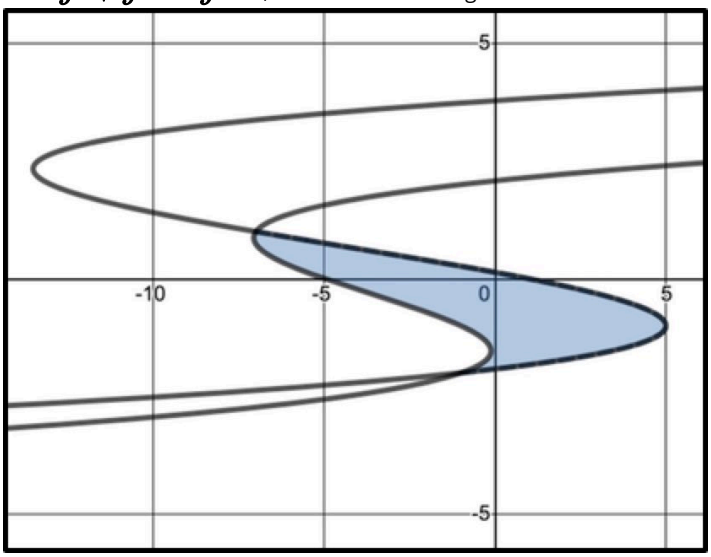
Using technology, calculate the approximate shaded area between the curves $y = \cos(\pi x) + \ln(x)$ and $y = \sin(\pi x) + \ln(x) - \frac{x}{\pi}$, as shown in the diagram.



Question #22

Reference Q.50239

Using algebraic techniques, calculate the exact value of the shaded area between the curves $x = y^3 - 2y^2 - 7y + 1$ and $x = y^3 + y^2 - 4y - 5$, as shown in the diagram.



Question #23

Reference Q.50203

Using technology, calculate the approximate shaded area between the curves $x = 4y^{\frac{1}{3}}$ and $x = 5y^{\frac{1}{5}}$, as shown in the diagram, by integrating with respect to y.