Calculus

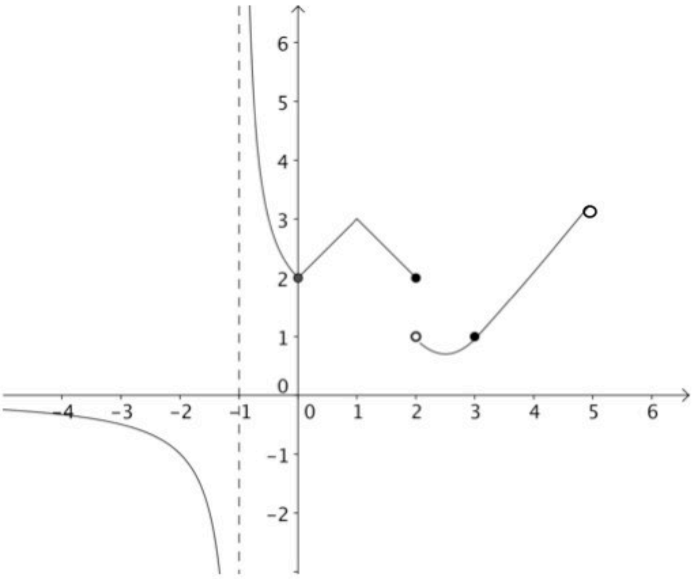
Chapter 2: Limits and Continuity

Lesson 1: Introduction to Limits

Question #1

Reference Q.19085

For the following function, $f(x)$, evaluate the following limits:



a. $\lim\limits_{x \to -1^-} f(x)$

b. $\lim\limits_{x \to -1^+} f(x)$

c. $\lim\limits_{x \to -1} f(x)$

d. $\lim\limits_{x \to 0^-} f(x)$

e. $\lim\limits_{x \to 0^+} f(x)$

f. $\lim\limits_{x \to 0} f(x)$

g. $\lim\limits_{x \to 1^-} f(x)$

h. $\lim\limits_{x \to 1^+} f(x)$

i. $\lim\limits_{x \to 1} f(x)$

j. $\lim\limits_{x \to 2^-} f(x)$

k. $\lim\limits_{x \to 2^+} f(x)$

l. $\lim\limits_{x \to 2} f(x)$

m. $\lim\limits_{x \to 3^-} f(x)$

n. $\lim\limits_{x \to 3^+} f(x)$

o. $\lim\limits_{x \to 3} f(x)$

p. $\lim\limits_{x \to 5^-} f(x)$

q. $\lim\limits_{x \to 5^+} f(x)$

Question #2

Reference Q.151

Consider the piecewise function

$$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1} & x \neq 1 \\ 3 & x = 1 \end{cases}$$

a. a) Graph the function on the domain [-3, 3].

b. b) Find the $\lim\limits_{x \to 1^-} f(x)$.

c. c) Find the $\lim\limits_{x \to 1^+} f(x)$.

d. d) Find the $\lim\limits_{x \to 1} f(x)$.

e. e) Find $f(1)$.

Question #3

Reference Q.153

Sketch a possible graph of a function that has domain $-3 \leq x \leq 5$, has zeros (intersects the x-axis) at -2, and 1, and the $\lim\limits_{x \to -2} f(x) = 5$, and $\lim\limits_{x \to 1} f(x) = DNE$

Possible answer:

Question #4

Reference Q.154

Sketch a possible graph of a function that has domain $x \leq 0$, but not $x = -2$, goes through the points (-4,2) and (0,2), and has $\lim\limits_{x \to -2} f(x) = \infty$.

Possible answer:

Question #5

Reference Q.155

Sketch a possible graph of a function that has zeros of -2, and 1, and $\lim\limits_{x \to -2^-} f(x) = -\infty$, but $\lim\limits_{x \to -2^+} f(x) = \infty$, and $\lim\limits_{x \to 1} f(x) = 3$

Possible answer:

Question #6

Reference Q.156

Estimate the value of $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$ by trying values very close to 0 (from both the right and the left, obviously, since it's not a one-sided limit). Once you've made a reasonable estimate, check it by actually graphing the function near that point.

Note: The limit value converges to "e"

Question #7

Reference Q.157

Estimate the value of $\lim_{x\to 2^-} \frac{3x+6}{3x^3-24}$ by trying values very close to 2 (from the left only, of course). Once you've made a reasonable estimate, check it by actually graphing the function near that point.
Estimation:
Graph:

Question #8

Reference Q.158

For the function $f(x) = 3x^2$, approximate the slope of the tangent line at (1,3) by doing the following:

a. Make a graph of f(x) and plot a point at $x = a$, somewhere nearby the point (1,3).

b. Write the slope of the secant line m_{sec} that joins the point (1,3) and the point (a, f(a)) in terms of a, and simplify.

c. Take the $\lim_{a\to 1}\ m_{sec}$ to determine the actual slope of the tangent line at (1,3).

Question #9

Reference Q.159

For the function $f(x) = -2x^2$, approximate the slope of the tangent line at (1,- 2) using the secant line approximation method (see question ID 158 for the individual steps).

Question #10

Reference Q.160

Evaluate the following limit:

$$\lim_{x\to 5}\ 3x$$

Question #11

Reference Q.161

Evaluate this limit:

$$\lim_{x\to \pi}\ 4x^2$$

Question #12

Reference Q.162

Evaluate the following limit:

$$\lim_{x\to 0} \frac{8}{x}$$

Question #13

Reference Q.163

Evaluate this limit:

$$\lim_{x\to 0} \frac{\cos(x)}{x}$$

Question #14

Reference Q.164

Evaluate this limit:

$$\lim_{x\to 0^+} \frac{5}{x^2}$$

Question #15

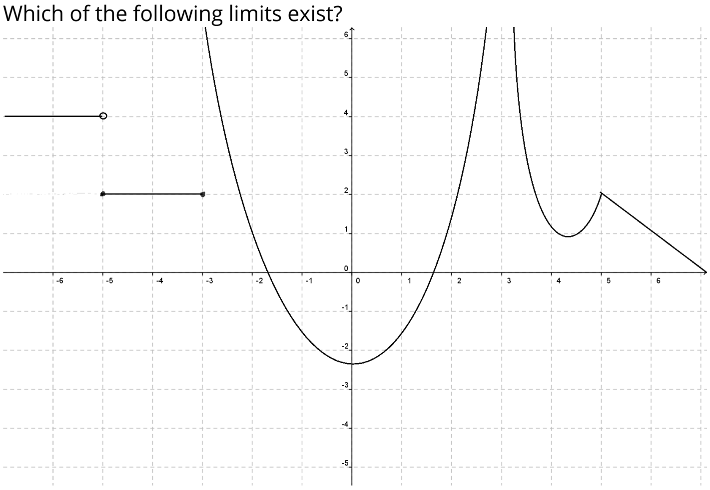
Reference Q.165

Evaluate the following limit:

$$\lim_{x\to 0^-} \frac{10}{x^3}$$

Question #16

Reference Q.3207
AP Prep:
Which of the following limits exist?



a. $\lim_{x\to -5^-} f(x)$
b. $\lim_{x\to -5^+} f(x)$
c. $\lim_{x\to -5} f(x)$
d. $\lim_{x\to -3^-} f(x)$
e. $\lim_{x\to -3^+} f(x)$
f. $\lim_{x\to -3} f(x)$
g. $\lim_{x\to 0^-} f(x)$
h. $\lim_{x\to 0^+} f(x)$
i. $\lim_{x\to 0} f(x)$
j. $\lim_{x\to 3^-} f(x)$
k. $\lim_{x\to 3^+} f(x)$
l. $\lim_{x\to 3} f(x)$
m. $\lim_{x\to 5^-} f(x)$
n. $\lim_{x\to 5^+} f(x)$
o. $\lim_{x\to 5} f(x)$

Question #17

Reference Q.9149
AP Prep:
Consider the function:
$$f(x) = \begin{cases} x^2+2 & x<3 \\ x & x\geq 3 \end{cases}$$
Which of the following are true?

a. $f(3) = 11$
b. $\lim_{x\to 3} f(x) = 3$
c. $\lim_{x\to 3^-} f(x)$ exists.
d. $\lim_{x\to 3} f(x)$ exists.
e. $\lim_{x\to 3^+} f(x) = 3$
f. $\lim_{x\to 3^+} f(x) = f(3)$

Question #18

Reference Q.3208
[Try This!]:
What is the formal definition of a limit of a real function? This is also known as the epsilon-delta definition of a limit (hint: wikipedia!). Once you've got the definition, try to figure out what it means, and see if you can use it to show that $\lim_{x\to 2} 2x+2=6$.

Question #19

Reference Q.9130
[Try This!]:
What is the formal definition of a limit of a real function? This is also known as the epsilon-delta definition of a limit (hint: wikipedia!). Once you've got the definition, try to figure out what it means, and see if you can use it to show that $\lim_{x\to 1} 3x-2=1$.

Question #20

Reference Q.9148
[Try This!]:
$\lim_{x\to c} f(x) = L$ is equivalent to:
(a) For all δ, there is an ϵ such that if $|x-c|<\delta$, then $|f(x)-L|<\epsilon$.
(b) For all ϵ, there is a δ such that if $|x-c|>\delta$, then $|f(x)-L|<\epsilon$.
(c) For all ϵ, there is a δ such that if $|x-c|<\delta$, then $|f(x)-L|<\epsilon$.
(d) For all δ, there is an ϵ such that if $|f(x)-L|<\epsilon$, then $|x-c|<\delta$.