Calculus

Chapter 3: Differentiation

Lesson 1: The Derivative

Question #1

Reference Q.249

Use the definition of the derivative to find $f'(a)$ for:

$f(x) = 3x^2, \quad a = 1$

Question #2

Reference Q.250

Use the definition of the derivative to find $f'(a)$ for:

$f(x) = \sqrt{x}; \quad a = 1$

Question #3

Reference Q.251

Use the definition of the derivative to find $\frac{dy}{dx}|_{x=0}$ for $y = 2x^3$.

Question #4

Reference Q.252

Use the definition of the derivative to find $\frac{dy}{dx}$ given that $y = \frac{2}{x}$.

Question #5

Reference Q.253

There is an alternative option for calculating the derivative at a certain point:

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$,

which many calculus courses will also teach (and hence, you must know (could be on the quiz and test) -- though you may even like it better), which also finds the derivative at x=a. Use this alternative to determine the following:

Find $f'(3)$ when $f(x) = 5x^2$.

Question #6

Reference Q.254

There is an alternative option for calculating the derivative at a certain point:

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

which many calculus courses will also teach (and hence, you must know (could be on the quiz and test) -- though you may even like it better), which also finds the derivative at x=a. Use this alternative to determine the following:

Find $f'(1)$ when $f(x) = \frac{4}{x}$.

Question #7

Reference Q.255

There is an alternative option for calculating the derivative at a certain point:

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$,

which many calculus courses will also teach (and hence, \underbar{you must know} (could be on the quiz and test) -- though you may even like it better), which also finds the derivative at x = a. Use this alternative to determine the following:

Find $f'(a)$ when $f(x) = \sqrt{x+2}$.

Question #8

Reference Q.256

A function goes through the point $(-5, 1)$ and $\frac{dy}{dx}|_{x=-5} = 7.$

What is the equation of the tangent line to the function at $x = -5$?

Question #9

Reference Q.276

A curve goes through the point $(\pi, 2)$ and has $\frac{dy}{dx}|_{x=\pi} = 1.$

Find the equation of the line tangent to the curve at $x = \pi$

Question #10

Reference Q.10680

Using terms like “secant line” and “tangent line”, explain how evaluating $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ will result in the value of the derivative of $f(x)$ at $x = a$.

Question #11

Reference Q.257

If $f'(a) = \lim_{h \to 0} \frac{\sqrt{4+h}-2}{h}$, what is $f(x)$ and what is a?

Question #12

Reference Q.258

If $f'(a) = \lim_{x \to 2} \frac{\sin x^2 - sin(4)}{x-2}$, what is $f(x)$ and what is a?

Question #13

Reference Q.259

Use the definition of the derivative to evaluate $f'(5)$ if $f(x) = 3 - x^3$.

Question #14

Reference Q.261

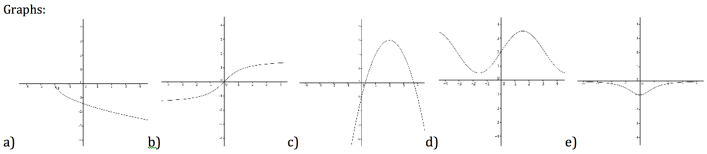
Sketch a graph where $f(2) = 3, \quad f'(2) = 0, \quad f'(x) > 0$ when $x < 2$ and $f'(x) < 0$ when $x > 2$.

Question #15

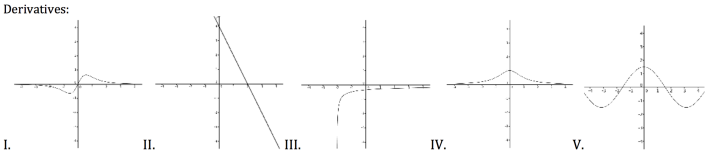
Reference Q.262

Match the graphs of the functions (a to e) with the graphs of their derivatives (I to V):

Graphs:



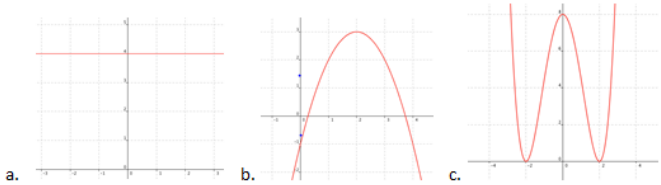
Derivatives:



Question #16

Reference Q.263

Sketch the graph of the derivative of the functions graphed below.



Question #17

Reference Q.264

Show that $f(x) = x^{\frac{1}{5}}$, whose derivative is $f'(x) = \frac{1}{5}x^{\frac{-4}{5}}$, is continuous at $x = 0$ but is not differentiable at that point.

Question #18

Reference Q.265

Show that $f(x) = \begin{cases} x^2 + 3, & x \leq 2 \\ 2x + 3, & x > 2 \end{cases}$ is continuous at $x = 2$ but not differentiable at $x = 2$.

Question #19

Reference Q.9156

For the function $f(x) = |x + 1| - 5$, find $f'(-1)$.

Question #20

Reference Q.9157

Suppose $\lim_{h \to 0} \frac{f(h) - f(0)}{h} = e$. Which of the following are true?

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f'(x)$ is continuous at $x = 0$

(d) $f'(x)$ is differentiable at $x = 0$

Question #21

Reference Q.9158

Let $f(x) = \sin 2^x$. Which of the following is $f'(x)$?

(a) $\lim_{h \to 0} \dfrac{\sin 2^{x+h}}{h}$

(b) $\lim_{h \to 0} \dfrac{\sin 2^x 2^h - \sin 2^x}{h}$

(c) $\lim_{h \to 0} \dfrac{\sin 2^h - \sin 2^x}{h}$

(d) $\lim_{h \to 0} \dfrac{\sin (2^x + h) - \sin 2^x}{h}$

Question #22

Reference Q.9159

Suppose that $f(x)$ is an odd function, and that $f'(2) = 4$. What is the value of $f'(-2)$?