Calculus

Chapter 5: Integration

Lesson 2: Introduction to the Definite Integral

Question #1

Reference Q.390

Express the given integral as the limit of a Riemann sum but do not evaluate:

$\int_1^3 (x+1)\,dx$

Question #2

Reference Q.391

Express the given integral as the limit of a Riemann sum but do not evaluate:

$\int_0^2 \frac{x^2}{x+1}dx$

Question #3

Reference Q.392

Translate this expression into an integral, but do not evaluate it.

$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 2\cos(x_k^*)\Delta x_k \qquad a=3, b=7$

Question #4

Reference Q.393

Translate this expression into an integral, but do not evaluate it.

$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (5x_k^* + 2)\,\triangle x_k;\ a=-3;\ b=2$

Question #5

Reference Q.394

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_1^2 2dx$

a. Sketch

b. evaluate

Question #6

Reference Q.395

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_0^2 x dx$

a. sketch

b. evaluate

Question #7

Reference Q.396

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_{-2}^0 (2x+6)\,dx$

Question #8

Reference Q.397

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_{-1}^2 (-2x-3)\,dx$

Question #9

Reference Q.398

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_{-2}^2 |x|\,dx$

a. sketch

b. evaluate

Question #10

Reference Q.399

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx$

a. sketch

b. evaluate

Question #11

Reference Q.400

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_0^5 |2x-6|\ dx$

Question #12

Reference Q.401

Sketch the region indicated by the following integral and evaluate it using simple geometry.

$\int_0^2 \sqrt{4-x^2}dx$

a. sketch

b. evaluate

Question #13

Reference Q.402

For

$$f(x)=\begin{cases}-x+1, & \text{if } x\le 0\\ x+1, & \text{if } x>0\end{cases}$$

evaluate the following integral:

$\int_{-3}^2 f(x)\,dx$

Question #14

Reference Q.403

For $\begin{cases}-x+1. & \text{if } x\le -1\\ 4x+6, & \text{if } x>-1\end{cases}$, evaluate the following integral:

$\int_{-4}^{-1} f(x)\,dx$

Question #15

Reference Q.404

For $f(x)=\left\{\begin{matrix}-x+1, & x\le 0\\ x+1, & x>0\end{matrix}\right\}$, evaluate the following integral:

$\int_0^4 f(x)\,dx$

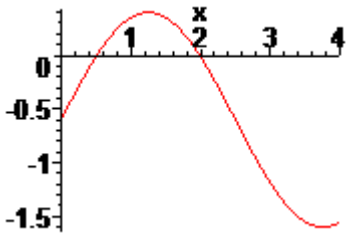
Question #16

Reference Q.405

For the graph of

$\int_0^{0.5} f(x)\,dx=-0.15,\quad \int_{0.5}^2 f(x)\,dx=0.39,\quad \int_2^4 f(x)\,dx=-2.09$

find



a. integral from 0 to 2

b. integral from 0.5 to 4

c. integral from 0 to 4

Question #17

Reference Q.406

Find $\int_{-1}^4 (f(x)-3g(x))\,dx$ if $\int_{-1}^4 f(x)\,dx=4$ and if $\int_{-1}^4 g(x)\,dx=-3$

Question #18

Reference Q.407

Find $\int_2^7 f(x)\,dx$ if $\int_{-3}^2 f(x)\,dx=5$ and if $\int_{-3}^7 f(x)\,dx=3$

Question #19

Reference Q.408

Evaluate the following integral by interpreting it geometrically:

$\int_{-4}^3 2x-5\ dx$

Question #20

Reference Q.409

Evaluate the following integral by interpreting it geometrically:

$\int_0^2 x+\sqrt{4-x^2}dx$

Question #21

Reference Q.410

Evaluate the following integral by interpreting it geometrically:

$\int_{-3}^2 1+|2x+1|\ dx$

Question #22

Reference Q.411

Evaluate the following limit by interpreting it geometrically:

$\lim_{max\triangle x_k \to 0} \sum_{k=1}^{n} \left(3 - 2x_k^*\right) \triangle x_k$ over the interval [3, 7].

Question #23

Reference Q.9223

AP Prep:

Which of the following is equal to $\int_0^{\frac{3\pi}{2}} \sin\left(\frac{x}{2}\right) dx$?

(A) $\int_0^{\frac{3\pi}{4}} \cos x \, dx$

(B) $\int_0^{\frac{3\pi}{2}} \cos\left(\frac{x}{2}\right) dx$

(C) $\int_{-\pi}^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx$

(D) $\int_{\frac{\pi}{2}}^{\frac{5\pi}{4}} \cos x \, dx$

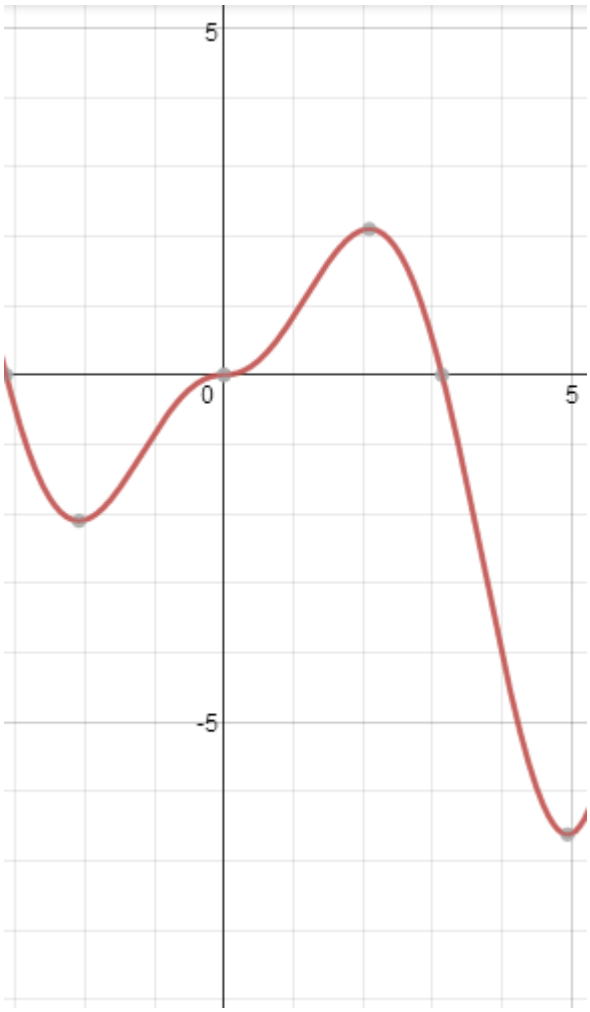
Sketch the sine and cosine functions and solve this geometrically.

Question #24

Reference Q.9224

AP Prep:

For the function $f(x)$ below, write an expression of the form $\int_b^a f(x) \, dx$ that gives the maximum (positive) area between the curve and the x-axis.

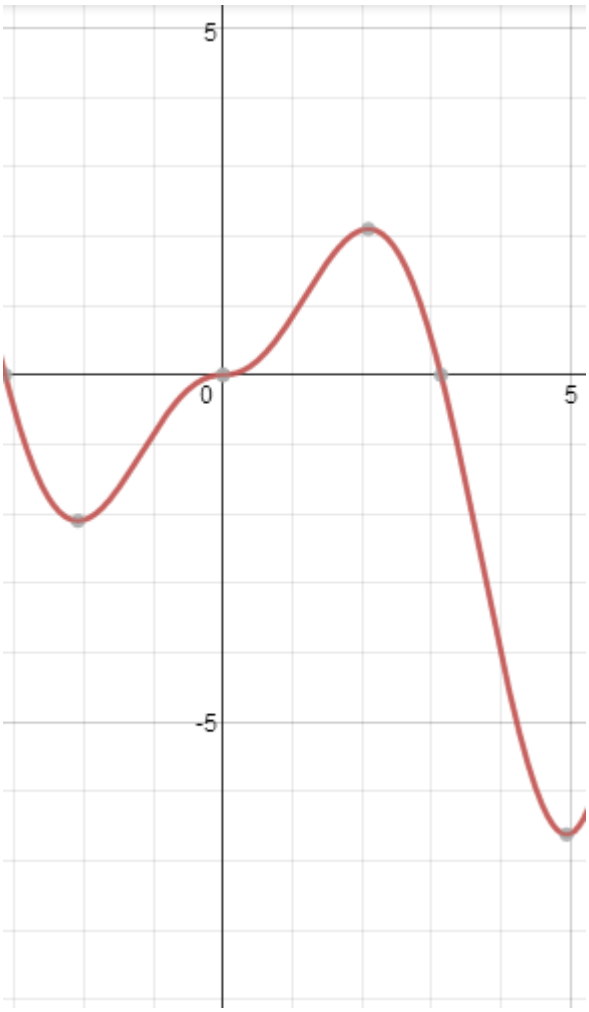


Question #25

Reference Q.9225

AP Prep:

For the function $f(x)$ below, write an expression of the form $\int_b^a f(x)\,dx$ that gives the minimum (positive) area between the curve and the x-axis.



Question #26

Reference Q.9226

AP Prep:

$\int_2^5 \frac{x}{e^x}\,dx$ is equivalent to which of the following, where $f(x) = \frac{x}{e^x}$?

(A) $\lim_{n\to\infty} f\left(2+\frac{3}{n}\right)\left(\frac{3}{n}\right)$

(B) $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\frac{3}{n}i\right)\left(\frac{3}{n}\right)$

(C) $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(2i+\frac{3}{n}\right)\left(\frac{3}{n}\right)$

(D) $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\frac{3}{n}\right)\left(\frac{3}{n}i\right)$