Calculus

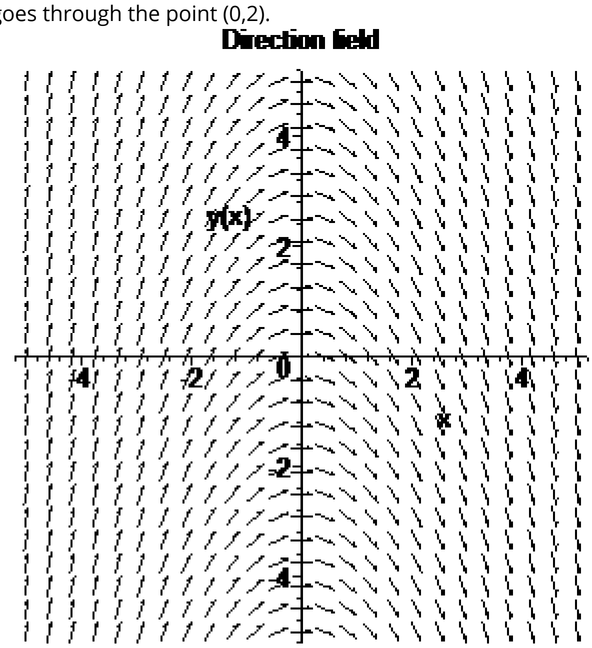
Chapter 7: Differential Equations and More Riemann Sums

Lesson 2: Initial Value Problems, Slope Fields, and Euler's Method

Question #1

Reference Q.687

Sketch some of the possible solutions for the following slope field of some differential equation -- and in particular, sketch the curve that goes through the point (0,2).

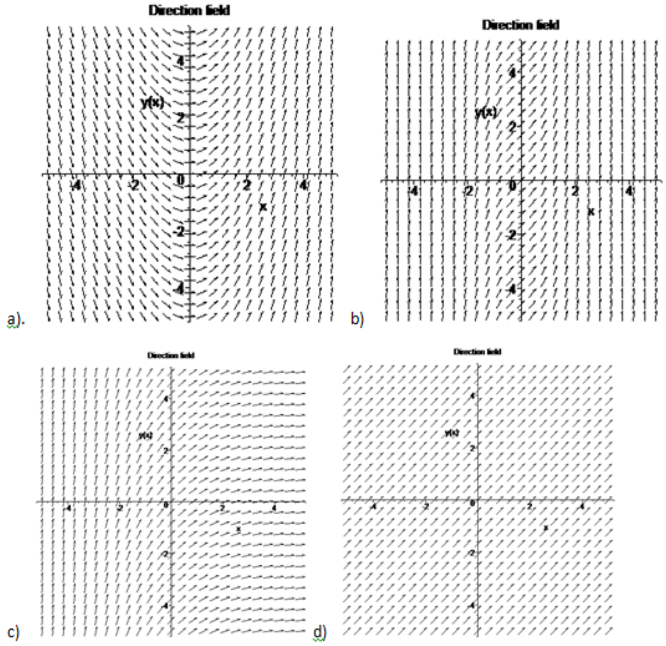


Question #2

Reference Q.688

Match the following differential equations below to their slope field.

1. $\frac{dy}{dx} = e^{-\frac{x}{2}}$
2. $\frac{dy}{dx} = x$
3. $\frac{dy}{dx} = 1$
4. $\frac{dy}{dx} = 1 + x^2$



Question #3

Reference Q.689

Sketch the slope field for $y' = \frac{3x}{y}$ where x=[-3..3] and $y = [-3..\ 3]$

Question #4

Reference Q.690

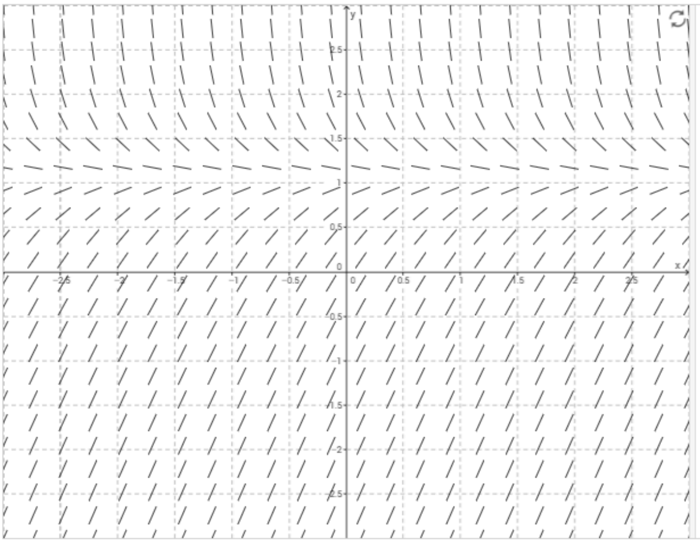
Sketch the slope field for

$y' = 2 - 2y\ at\ x = [-3..3]\ \ and\ y = [-3..3]$

Question #5

Reference Q.691

For the slope field of $y' = 3 - e^{y}$ shown below



a. a) sketch a graph of the solution that satisfies $y(0) = 0$.

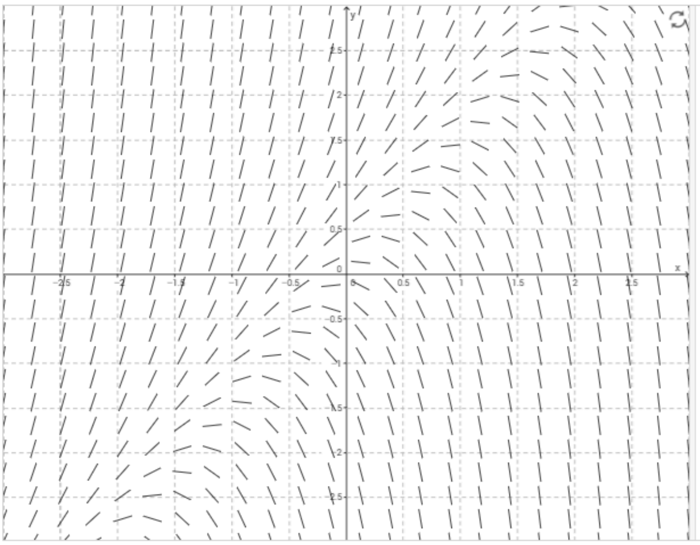
b. b) sketch a graph of the solution that satisfies $y(-1) = 0$.

c. c) Guess the value of $\lim_{x \to \infty} y(x)$ (Note: You could solve for y(x) to verify!)

Question #6

Reference Q.692

For the slope field of $y' = 2y - 3x$ shown below



a. a) sketch a graph of the solution that satisfies $y(2) = 0$.

b. b) sketch a graph of the solution that satisfies $y(-2) = 0$.

Question #7

Reference Q.693

For each of the examples below, match each equation to the corresponding slope field giving reasons why it is true:

a) $y' = \frac{2}{x}$

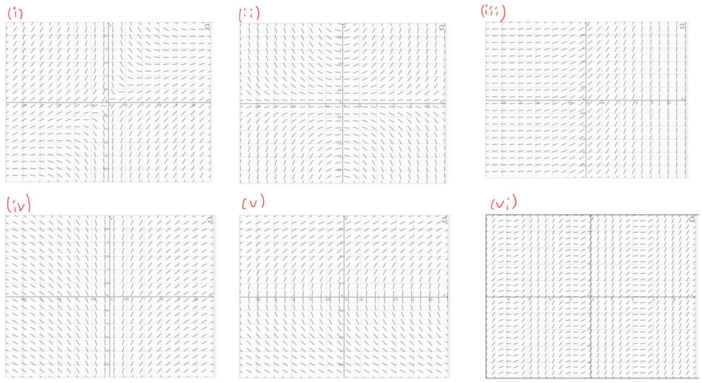
b) $y' = \frac{2}{y}$

c) $y' = \frac{x-y}{x}$

d) $y' = xy$

e) $y' = e^{x}$

f) $y' = 1 + \sin x$



Question #8

Reference Q.694

For the differential equation $\frac{dy}{dx} = x - 2y$ with the initial condition $y(0) = 1$, and $\Delta x = 0.5$, use Euler's Method to estimate the y-value at x=2.

Question #9

Reference Q.695

For the differential equation $\frac{dy}{dx} = x^2 - x - y$ with the initial condition (-1, 0), use Euler's Method to approximate the solution curve. Use $\triangle x = 0.5$, and make a rough sketch of the curve.

Question #10

Reference Q.696

For the differential equation $\frac{dy}{dx} = -2xy$ with the initial condition (-1, 1), use Euler's Method to approximate the solution curve (with $\triangle x = 0.5$). Then find the solution algebraically; sketch both the approximation and the exact solution on the same set of axes.

Question #11

Reference Q.9270

If $\dfrac{dy}{dx} = y\csc^2 x$ and when $x = \dfrac{\pi}{2}$, $y = 1$, find y.

Question #12

Reference Q.9271

Solve the initial value problem: $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{y^3}}$, $y(4) = 9$.

Question #13

Reference Q.9272

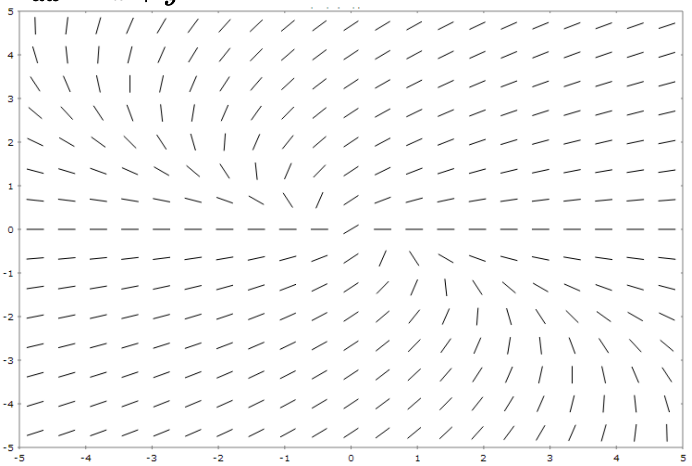
Which of the following differential equations could represent the slope field pictured bellow?

(a) $\dfrac{dy}{dx} = \dfrac{y}{x}$

(b) $\dfrac{dy}{dx} = \dfrac{x}{y}$

(c) $\dfrac{dy}{dx} = \dfrac{y}{x+y}$

(d) $\dfrac{dy}{dx} = \dfrac{y-x}{x+y}$



Question #14

Reference Q.9273

Which of the following could be the solution for $y(0) = 1$ for the slope field pictured below?

(a) $y = \dfrac{1}{x}$

(b) $y = \dfrac{1}{x^2+1}$

(c) $y = \dfrac{1}{x^3+1}$

(d) $y = \dfrac{1}{x^2}$