

Calculus

Chapter 2: Limits and Continuity

Lesson 2: Properties of Limits

Question #1

Reference Q.172

Evaluate $\lim_{x \rightarrow 5^-} \frac{x^2}{x-5}$

Question #2

Reference Q.173

Evaluate $\lim_{x \rightarrow 5} \frac{x^2}{x-5}$

Question #3

Reference Q.174

Evaluate $\lim_{m \rightarrow 3^-} \frac{m^2}{m^2-9}$

Question #4

Reference Q.175

Evaluate $\lim_{m \rightarrow 3} \frac{m-3}{m^2-9}$

Question #5

Reference Q.176

Evaluate $\lim_{m \rightarrow 1} \frac{m^2-1}{m^4-1}$

Question #6

Reference Q.177

Evaluate $\lim_{x \rightarrow 3} \frac{4-x}{3x^2-2x-21}$

Question #7

Reference Q.178

Evaluate $\lim_{x \rightarrow -4} \frac{x+4}{|x+4|}$

Question #8

Reference Q.179

Evaluate $\lim_{m \rightarrow 9} \frac{9m-m^2}{3-\sqrt{m}}$

Question #9

Reference Q.180

Show that $\lim_{x \rightarrow 2} \frac{x^2-4}{\sqrt{x^2+12}-4} = 8$.

Question #10

Reference Q.181

A function is defined as $f(x) = \frac{x-2}{x^2-4}$.

Graph the function close to the value of $x = 2$. Use your graphing calculator to zoom in to find the value of $\lim_{x \rightarrow 2} f(x)$. Use algebraic methods to solve $\lim_{x \rightarrow 2} f(x)$ and prove your result.

- Graph:
- Value of $\lim_{x \rightarrow 2} f(x)$:

Question #11

Reference Q.182

A piecewise function has the following form:

$$f(x) = \begin{cases} 2x+1, & x \leq 2 \\ \sqrt{9x+7}, & x > 2 \end{cases}$$

Find:

- $\lim_{x \rightarrow 2^-} f(x)$
- $\lim_{x \rightarrow 2^+} f(x)$
- $\lim_{x \rightarrow 2} f(x)$

Question #12

Reference Q.183

We know the following statement is true (Sum and Difference Limit Laws):

If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x).$$

Decide whether

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{2}{x} \right) = \lim_{x \rightarrow 0} \frac{1}{x} - \lim_{x \rightarrow 0} \frac{2}{x}$$

is true or false. If it is true, find the limit. If not, explain why it is false.

Question #13: Proof: Properties of Limits #1

Reference Q.64782

If $\lim_{x \rightarrow c} f(x) = A$ and $\lim_{x \rightarrow c} g(x) = B$, prove

$$\lim_{x \rightarrow c} (f(x) + g(x)) = A + B.$$

Question #16

Reference Q.9132

Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin^2 \left(x - \frac{\pi}{2} \right)}{x - \frac{\pi}{2}}$

Question #17

Reference Q.9150

Evaluate the following limit:

$$\lim_{x \rightarrow 3} \frac{3x^2 - 3x - 18}{2x^2 + 4x - 30}$$

Question #14: Proof: Properties of Limits #2

Reference Q.64785

If $\lim_{x \rightarrow c} f(x) = A$ and $\lim_{x \rightarrow c} g(x) = B$, prove

$$\lim_{x \rightarrow c} (f(x) - g(x)) = A - B.$$

Question #15: Proof: Properties of Limits #3

Reference Q.64788

If $\lim_{x \rightarrow c} f(x) = A$ and $\lim_{x \rightarrow c} g(x) = B$, prove

$$\lim_{x \rightarrow c} (2f(x) + (g(x))^2) = 2A + B^2.$$

Question #18

Reference Q.9151

Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\ln x}{x}$$

(We restrict the domain to $x \in (0, \infty)$).

Question #19

Reference Q.9152

For what value(s) of a is the following limit equal to 0?

$$\lim_{x \rightarrow 1} \frac{x^2 - a^2}{x^2 + ax + x + a}$$