

# Calculus

## Chapter 3: Differentiation

### Lesson 3: Trigonometric Derivatives and The Chain Rule

#### Question #1

Reference Q.301

$f(x) = 5 + 3 \cos x$ . What is  $f'(x)$ ?

#### Question #2

Reference Q.302

$y = 4x^3 \sin x$ . What is  $\frac{dy}{dx}$ ?

#### Question #3

Reference Q.303

$f(x) = \frac{4 \sin(x)}{1 - \cos x}$ . What is  $f'(x)$ ?

#### Question #4

Reference Q.304

$y = 3 \csc x - \cot x$ . What is  $\frac{dy}{dx}$ ?

#### Question #5

Reference Q.305

$f(x) = 2 \sec x + \sqrt{x} \tan x$ . What is  $f'(x)$ ?

#### Question #6

Reference Q.306

$y = \frac{3 \csc x}{3 + \tan x}$ . What is  $\frac{dy}{dx}$ ?

#### Question #7

Reference Q.307

$f(x) = \csc x \cot x$ . What is  $f'(x)$ ?

#### Question #8

Reference Q.272

Find  $\frac{dy}{dx}$  when  $y = 2(3 - 4x^3)^2$ ?

#### Question #9

Reference Q.308

$f(x) = \cos^2(x) - \sin^2(x)$ . What is  $f'(x)$ ?

#### Question #10

Reference Q.309

Find  $\frac{d^2y}{dx^2}$ :  $y = \cot(x)$

#### Question #11

Reference Q.310

Find the equation of the tangent line to the graph of  $y = 2 \cos x$  at  $x = \frac{\pi}{2}$ .

#### Question #12

Reference Q.311

A rocket is launched into the air from a platform which is 5 miles away from a crowd of people watching. If the height of the rocket is given by  $h$ , what is the rate of change of the height with respect to the angle,  $x$ , (i.e. calculate  $\frac{dh}{dx}$ )

#### Question #13

Reference Q.312

Try to determine the pattern followed by taking successive derivatives of the following trig function, and then calculate the actual derivative:

(a)  $\frac{d^{1001}}{dx^{1001}} \cos x$

(b)  $\frac{d^{1001}}{dx^{1001}} \sin x$

#### Question #14

Reference Q.313

Try to determine the pattern followed by taking successive derivatives of the following trig function, and then calculate the actual derivative:

$\frac{d^{1001}}{dx^{1001}} \cos x + e^{-x}$

### Question #15

Reference Q.314

State the Chain Rule in Newtonian Form and in Leibniz Form.

### Question #16

Reference Q.315

If  $f(x) = x^2$  and  $g(x) = 3x + 2$  find the derivative of  $f(g(x))$

### Question #17

Reference Q.316

If  $f(x) = \sin(x)$  and  $g(x) = -\frac{x}{4}$  find the derivative of  $f(g(x))$

### Question #18

Reference Q.317

Use the following table of values to find:

$x$	$a(x)$	$b(x)$	$a'(x)$	$b'(x)$
1	3	3	2	-2
3	1	-2	5	-1

a. a)  $A'(1)$  where  $A(x) = a(b(x))$

b. b)  $B'(1)$  where  $B(x) = b(a(x))$

### Question #19

Reference Q.318

What is  $f'(x)$  if  $f(x) = (4x - 3)^8$ ?

### Question #20

Reference Q.319

What is  $f'(x)$  when  $f(x) = \cos^3 x$ ?

### Question #21

Reference Q.320

What is  $f'(x)$  when  $f(x) = \cos(x^3)$ ?

### Question #22

Reference Q.321

What is  $f'(x)$  when  $f(x) = \sqrt{5 - x}$ ?

### Question #23

Reference Q.322

What is  $f'(x)$  when  $f(x) = 2 \csc^4 x^3$ ?

### Question #24

Reference Q.323

What is  $f'(x)$  when  $f(x) = 2(\csc x^3)^4$ ?

### Question #25

Reference Q.324

What is  $\frac{dy}{dx}$  when  $y = \sin(\sin(x))$ ?

### Question #26

Reference Q.325

Find  $\frac{dy}{dx}$  when  $y = x^2 \sec^2\left(\frac{x}{2}\right)$ .

### Question #27

Reference Q.326

Find the tangent to the graph of  $y = \sin(x^2) + 2x$  at  $x = \pi$ .

### Question #28

Reference Q.327

Find  $\frac{dy}{d\theta}$  for the equation  $y = (\tan(\pi - 2\theta))^4$ .

### Question #29

Reference Q.328

For the following table of values, find  $h'(1)$ :

a.  $h(x) = f(x) + g(x)$

b.  $h(x) = f(g(x))$

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	4	5
3	6	7	8	9

🔍 **Question #30**

Reference Q.329

Using the following table, find  $h'(2)$ , where

(a)  $h(x) = \frac{f(x)}{g(x)}$

(b)  $h(x) = g(f^2(x))$

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	5	7	9
9	11	13	15	17

🔍 **Question #32**

Reference Q.331

Find a formula for  $\frac{d}{dx}u(v(w(x)))$

🔍 **Question #33**

Reference Q.9137

Let

$$f(x) = g(x) \cos(h(x)), \quad g(x) = 1 + \frac{1}{x}, \quad h(x) = \sqrt{p(x)}, \quad p(x) = x^3$$

. Find  $f'(x)$ .

🔍 **Question #31**

Reference Q.330

Prove that the derivative of  $\cos x = -\sin x$ .

🔍 **Question #34**

Reference Q.9162

Find  $\frac{dy^2}{d^2x}$  if  $y = \cos(ax + b)$ .

🔍 **Question #35**

Reference Q.9163

Find  $\frac{dy}{dx}$  if  $y = \sin^2(x^2 + 2)$