Calculus

Chapter 3: Differentiation

Lesson 4: Derivatives of Exponential, Logarithmic, and Inverse Trig Functions

Question #1

Reference Q.332

Find $f'(x)$ when $f(x) = \ln 3x$.

Question #2

Reference Q.333

Find $f'(x)$ when $f(x) = \ln x^3$.

Question #3

Reference Q.334

Find $f'(x)$ when $f(x) = \ln\left(\frac{3x}{2+x^2}\right)$.

Question #4

Reference Q.335

Find $f'(x)$ when $f(x) = \log\left(1 - x^2\right)$?

Question #5

Reference Q.336

Find $f'(x)$ when $f(x) = (\ln x)^2$.

Question #6

Reference Q.337

Find $f'(x)$ when $f(x) = \log_3 \sqrt{x}$.

Question #7

Reference Q.338

Find $f'(x)$ when $f(x) = 3x^2 \ln x$.

Question #8

Reference Q.339

Find $f'(x)$ when $f(x) = \sqrt[3]{\log x}$.

Question #9

Reference Q.340

Find $f'(x)$ when $f(x) = x^2 \log_3(x^2 + 3)$.

Question #10

Reference Q.341

Find $f'(x)$ when $f(x) = \frac{x^2}{\log_3(x^2+3)}$.

Question #11

Reference Q.342

Find $f'(x)$ when $f(x) = 3 \ln(\ln x)$.

Question #12

Reference Q.343

Find $f'(x)$ when $f(x) = \log_5(\cot x)$.

Question #13

Reference Q.344

Find $f'(x)$ when $f(x) = \sin(\ln x)$.

Question #14

Reference Q.345

Find $f'(x)$ when $f(x) = e^{8x}$.

Question #15

Reference Q.346

Find $f'(x)$ when $f(x) = 4^{3x+1}$.

Question #16

Reference Q.347

Find $f'(x)$ when $f(x) = e^{x^2-1}$.

Question #17

Reference Q.348

Find y' when $y = x^3 5^{x-1}$.

Question #18

Reference Q.349

Find $f'(x)$ when $f(x) = 3x2^{\sqrt{x}}$.

Question #19

Reference Q.350

Find $f'(x)$ when $f(x) = 2x^{e^{2x}}$

Question #20

Reference Q.355

Find the tangent line to the graph of $f(x) = 2(\ln x)^2$ at $x_0 = \frac{1}{e}$.

Question #21

Reference Q.356

AP PREP! For the following question, interpret the limit as a suitable derivative, and evaluate.

[That is, COMPARE it to one of our two definitions we have for the derivative (immediately below) and try to identify which function is being differentiated, and where.]

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ OR $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Evaluate:

$\lim_{h\to 0} \frac{e^{3+h}-e^3}{h}$

Question #22

Reference Q.357

AP PREP! For the following question, interpret the limit as a suitable derivative, and evaluate.

[That is, COMPARE it to one of our two definitions we have for the derivative (immediately below) and try to identify which function is being differentiated, and where.]

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ OR $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Evaluate:

$\lim_{x\to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}}$

Question #23

Reference Q.358

AP PREP: For the following question, interpret the limit as a suitable derivative, and evaluate.

[That is, COMPARE it to one of our two definitions we have for the derivative (immediately below) and try to identify which function is being differentiated, and where.]

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ OR $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Evaluate:

$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

Question #24

Reference Q.359

For the following question, interpret the limit as a suitable derivative, and evaluate.

[That is, COMPARE it to our two definitions we have for the derivative (immediately below) and try to identify which function is being differentiated, and where.]

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ OR $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Evaluate:

$\lim_{h\to 0} \frac{8^h-1}{h}$

Question #25

Reference Q.360

Memorize the derivatives of the six inverse trig functions.

Question #26

Reference Q.361

Find $\frac{dy}{dx}$ if $y = \cos^{-1} 2x$.

Question #27

Reference Q.362

Find $\frac{dy}{dx}$ if $y = \sin^{-1} \frac{3}{x}$.

Question #28

Reference Q.363

Find $\frac{dy}{dx}$ if $y = \tan^{-1} x^4$.

Question #29

Reference Q.364

Find $\frac{dy}{dx}$ if $y = \sec^{-1} x^3$.

Question #30

Reference Q.365

Find $\frac{dy}{dx}$ if $y = \csc^{-1} x^2$.

Question #31

Reference Q.366

Find $\frac{dy}{dx}$ if $y = \frac{3}{\sin^{-1} x}$.

Question #32

Reference Q.367

Find $\frac{dy}{dx}$ if $y = e^{3x} \sec^{-1} 2x$.

Question #33

Reference Q.50158

If $f(x) = -2x^3$, evaluate $(f^{-1})'(16)$ if it exists.

Question #34

Reference Q.50161

Suppose that $f(x)$ is decreasing on the interval $(-\infty, \infty)$. If $f(0) = 1$, and $f'(x) = -3x^2 - 2$, find $(f^{-1})'(1)$.

Question #35

Reference Q.50164

A table of values for $f(x)$ and $f'(x)$ is given below. Assume that $f(x)$ is decreasing and differentiable on the interval $(-\infty, \infty)$. Use the table to find the equation of the line tangent to $f^{-1}(x)$ at $f(x) = -6$.

x	-1	0	1	2
f(x)	-2	-6	-7	-11
f'(x)	-3	-5	-2	-4

Question #36

Reference Q.50167

Suppose f and g are inverse functions of each other and are differentiable at all $x \in \mathbb{R}$. If $f(2) = 2$ and $f'(2) = 3$, and $g(-1) = -2$, which of the following statements must be false?

a. $f(-2) = -1$

b. $g(2) = 2$

c. $g'(2) = \frac{1}{3}$

d. $f'(-1) = \frac{1}{g'(-2)}$

Question #37

Reference Q.50170

For all $x \in \mathbb{R}$, suppose $f(x)$ is differentiable, invertible, and $f'(x) \neq 0$. Prove $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ by differentiating the other form of invertibility (i.e. different than the one shown in the video), namely $f^{-1}(f(x)) = x$.

Hint: Use $y = f(x)$.

Question #38

Reference Q.368

Show that for any constants a and b that $y = ae^{bt}$ satisfies $\frac{dy}{dt} = by$.

Question #39

Reference Q.9139

Find $\frac{dy^n}{d^n x}$ for $y = e^{-nx}$. As $n \to \infty$, what is $\frac{dy^n}{d^n x}$?

Question #40

Reference Q.9164

Find $\frac{dy}{dx}$ if $y = \sin^{-1}(\ln x)$.

Question #41

Reference Q.9165

Find $\frac{dy}{dx}$ if $y = e^{\tan^{-1} x^2}$