Calculus

Chapter 6: Applications of Integrals

Lesson 5: Average Value of a Function and Rectilinear Motion Revisited

Question #1

Reference Q.646
Find x^* (as described by The Mean Value Theorem for Integrals) for the following function on the given interval:
$f(x) = 2e^x + 1,\ [-2, 3]$

Question #2

Reference Q.647
Find x^* (as described by The Mean Value Theorem for Integrals) for the following function on the given interval:
$f(x) = \cos x, \quad \left[0, \frac{\pi}{2}\right]$

Question #3

Reference Q.645
Find x^* (as described by The Mean Value Theorem for Integrals) for the following function on the given interval:
$f(x) = 3 \ln x$ on $[1, e]$
Note: You will need to use this fact: $\int \ln x \, dx = x \ln x - x + C$
(this is proved later in calculus)

Question #4

Reference Q.648
Find the average value of the following functions over the given interval.
$f(x) = 5x, \quad [2, 6]$

Question #5

Reference Q.649
Find the average value of the following functions over the given interval.
$f(x) = \cos x, \quad [-\frac{\pi}{2}, \frac{\pi}{2}]$

Question #6

Reference Q.650
Find the average value of the following functions over the given interval.
$f(x) = \sqrt{2x + 1}, \quad [0, 4]$

Question #7

Reference Q.651
Find the average value of the following functions over the given interval.
$f(x) = \csc x \cot x, \quad [\frac{\pi}{3}, \frac{\pi}{2}]$

Question #8

Reference Q.652
Find the average value of the following functions over the given interval.
$f(x) = \frac{3}{x} + 3e^x, \quad [e, 4]$

Question #9

Reference Q.653
Find the average value of the following functions over the given interval.
$f(x) = \frac{x}{1+4x^2}, \quad [0, 3]$

Question #10

Reference Q.654
Find the average value of the following functions over the given interval.
$f(x) = \dfrac{\cos x}{\sin^2 x}, \quad [\dfrac{\pi}{6}, \dfrac{\pi}{3}]$

Question #11

Reference Q.655
(Use a calculator to evaluate an integral !!)
Find the average value of the following functions over the given interval.
$f(x) = \sqrt{\ln(\sec(x^2))}, \quad [0, 1]$

Question #12

Reference Q.656

a. What's the average speed of an object travelling at $v(t) = 5t^2 + 7$ m/s over the time $3 \leq t \leq 5$?

b. at what exact time in that interval does the object travel at that average speed?

Question #13

Reference Q.657

Sally reasoned that if she drove 30 km/hr in one direction over a 180 km distance, and then returned travelling at 60 km/hr over the same distance, that her average speed must be half way between 30 and 60, which is 45 km/hr. Is she right? Why or why not?

Question #14

Reference Q.658

Find d(t) if $v(t) = 2t^2 - 3t \quad m/s\,, \;\; d(0) = 5.0\ m$

Question #15

Reference Q.659

Find d(t) if $a(t) = 2\cos(t)\,,\; v(0) = 0\frac{m}{s},\; d(0) = 0\ m$

Question #16

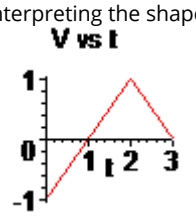
Reference Q.660

Find d(t) If a particle travels in a straight line according to $v(t) = 2t + 2$ m/s, and after a time of 5 seconds its position is at 3 meters.

Question #17

Reference Q.661

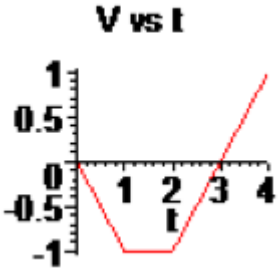
Given the following graph of velocity versus time, find the displacement and the distance traveled over $0 < t < 3$ sec, by interpreting the shape geometrically (using areas of common shapes).



Question #18

Reference Q.662

Given the following graph of velocity versus time, find the displacement and the distance traveled over 0<t<4



Question #19

Reference Q.663

A speedboat accelerates from rest at $3.2\frac{m}{s^2}$. After accelerating for $100\ m$, it shuts off the motor and slows down using water friction at an acceleration of $-4.6\ m/s^2$ until it's fully stopped. How far did the speedboat travel in total?

Question #20

Reference Q.664

A batter unintentionally hits a ball straight up into the air with an initial speed of $20\frac{m}{s}$ at an initial height of $1.2\ m$.

a. a) How long will it take the ball to reach its maximum height?

b. b) How high does it go?

c. c) How long will it take the ball to be caught by the catcher at $1.7\ m$ high?

d. d) What is the speed of the ball just before the catcher catches it?

Question #21

Reference Q.666

A police motorcycle accelerates at $4.0\frac{m}{s^2}$ from being stopped behind a billboard after a truck traveling at $80\frac{km}{hr}$ in a 50 km speed-zone. If the motorcycle begins its acceleration after the truck is 20 m away, and assuming the trucks speed is constant, how long will it take to catch up, and how far will the police officer have travelled to catch up?

Question #22

Reference Q.9254

$t = 0$, a particle starts at rest, but speeds up very quickly with an acceleration of $a(t) = 18t^2$ metres per second. How far does it get in the first second?

Question #23

Reference Q.9255

A particle with acceleration $a(t) = -(t-2)^2 + 1$ m/s^2 is travelling in a straight line. If it has reached a velocity of 3 m/s by the time the first second has passed - at that moment, how far is it from where it will be 5 seconds later (assume the particle's position at $t = 0$ is 0)?

Question #24

Reference Q.9256

A particle moving along a straight line has an acceleration of $a(t) = t^3 + t - 1$ m/s^2. In the first ten seconds, how many times does it change direction? Note, the particle started from rest.

Question #25

Reference Q.9257

A particle is accelerating from rest at $a(t) = t - e^{-t}$ m/s^2; how far does it get in ten seconds?