Calculus

Chapter 5: Integration

Lesson 6: Integration by Parts

Question #1

Reference Q.7884

Evaluate the following indefinite integral using integration by parts:

$$\int x \cos 4x \, dx$$

Question #2

Reference Q.7989

Evaluate the following indefinite integral using integration by parts:

$$\int \frac{x}{\sqrt{x-1}} \, dx$$

Question #3

Reference Q.7990

Evaluate the following indefinite integral using integration by parts:

$$\int x \ln x \, dx$$

Question #4

Reference Q.7993

Evaluate the following indefinite integral using integration by parts:

$$\int x^3 e^{8x} \, dx$$

Question #5

Reference Q.7992

Evaluate the following indefinite integral using integration by parts:

$$\int \ln x + x e^x \, dx$$

Question #6

Reference Q.7991

Evaluate the following indefinite integral using integration by parts:

$$\int e^{-x} \cos x \, dx$$

Question #7

Reference Q.8046

Evaluate the following definite integral using integration by parts:

$$\int_1^e x^2 \ln x \, dx$$

Question #8

Reference Q.7995

Evaluate the following definite integral using integration by parts:

$$\int_0^\pi e^x \cos x \, dx$$

Question #9

Reference Q.7994

Evaluate the following indefinite integral using integration by parts:

$$\int x^2 \sin 4x \, dx$$

Question #10

Reference Q.8047

Evaluate the following definite integral using integration by parts:

$$\int_0^\pi x \sin x \, dx$$

Question #11

Reference Q.9285

A continuous function, $f(x)$, with a continuous derivative between $x = 0$ and $x = 1$, has the following values: $f(0) = -2$, $f(1) = 2$. If $\int_0^1 f(x) \, dx = 3$, then what is the value of $\int_0^1 x f'(x) \, dx$?

Question #12

Reference Q.8048

Find the area under $f(x) = xe^x$ between $x = 0$ and $x = 4$

Question #13

Reference Q.9288

Find the volume of a solid created by rotating $f(x) = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ around the y-axis.

Question #14

Reference Q.8049

On a very bouncy trampoline, Sally continues to bounce higher and higher; her velocity can be modelled by the function $v(t) = t^2 \sin t$, where t is time in seconds and $f(t)$ is displacement in metres. What is the total distance she has bounced over after 5 seconds?

(Remember: total distance traversed = $\int_{t_i}^{t_f} |v(t)|\, dt$, or the unsigned area under the velocity function).

Question #15

Reference Q.8097

Fourier analysis is the mathematical way of representing any repeating function a sum of various terms. With this method we convert sound waves onto data to store in an mp3 player, music synthesizer, or CT scan or MRI. The theory says that there is a function $f(x)$ such that

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right),$$

where the a_n and b_n are given by the formulas:

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx, \qquad n = 0, 1, 2, \cdots$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx, \qquad n = 1, 2, 3, \cdots$$

Find a_0, a_n and b_n if $f(x) = x^2$ and $L = \pi$.

Question #16

Reference Q.8050

The captain of a starship has lost control of its warp drive; their velocity is increasing rapidly, $v(t) = 2te^t$ to be exact. Find the function $d(t)$ for their displacement from the point where the warp drive began to malfunction (i.e., $d(0) = 0$).

Question #17

Reference Q.8053

A small metal ball travelling on the end of a spring that is bouncing up and down will gradually slow down according to the equation $v(t) = 40\frac{cm}{s} e^{-3t} \sin 5t$ where t is in seconds. The displacement of the ball after 20 seconds is given by:

$$\int_0^{20} v(t)\, dt = \int_0^{20} 40\tfrac{cm}{s} e^{-3t} \sin 5t\, dt$$

Find the displacement of the ball after 20 s.

Question #18

Reference Q.8051

Write out the integration by parts formula for the integral $\int \frac{1}{x}\, dx$ and simplify without evaluating any integrals. The result might seem like it should be impossible - why is this not the case?

Question #19

Reference Q.8052

Derive the integration by parts formula.

Question #20

Reference Q.9340

Find $f(x)$:

$$\int x \csc^2 x\, dx = f(x) + \int \cot x\, dx$$

Question #21

Reference Q.9341

Find $f(x)$:

$$\int x^2 \ln x^2\, dx = \frac{x^3}{3} \ln x^2 - \int f(x)\, dx$$

Question #22

Reference Q.9356

Evaluate:

$$\int x \csc^2 x\, dx$$