Calculus

Chapter 4: Applications of Derivatives

Lesson 6: The Mean-Value Theorem and L'Hopital's Rule

Question #1

Reference Q.575

Use the Mean Value Theorem, if it applies (as in, ensure it meets the sufficient conditions (continuity, etc)), to find an x-value where the instantaneous slope there is the same as the average slope over the given interval.

$f(x) = x^2 + 4x - 12$ on $[-7,\ 0]$

Question #2

Reference Q.576

Use the Mean Value Theorem, if it applies (as in, ensure it meets the sufficient conditions (continuity, etc)), to find an x-value where the instantaneous slope there is the same as the average slope over the given interval.

$f(x) = \frac{x}{x-2}$ on $[-1, 3]$

Question #3

Reference Q.577

Use the Mean Value Theorem, if it applies (as in, ensure it meets the sufficient conditions (continuity, etc)), to find an x-value where the instantaneous slope there is the same as the average slope over the given interval.

$f(x) = 2x^2 - 5x$ on $[0, 6]$

Question #4

Reference Q.578

Use the Mean Value Theorem, if it applies (as in, ensure it meets the sufficient conditions (continuity, etc)), to find an x-value where the instantaneous slope there is the same as the average slope over the given interval.

$f(x) = \sqrt{x - x^2}$ on $[0, 1]$

Question #5

Reference Q.579

Use the Mean Value Theorem, if it applies (as in, ensure it meets the sufficient conditions (continuity, etc)), to find an x-value where the instantaneous slope there is the same as the average slope over the given interval.

$f(x) = \sqrt{9 - x^2}$ on $[-3,\ 2]$

Question #6

Reference Q.580

Suppose you and your fancy car drive 150 miles in 1.5 hours. However, you forgot that you left your smartphone on and because of that, your data was tracked by the police, and they issue you a speeding ticket based on the data. How did they know that you had to be speeding at some point?

(Assume the general maximum speed limit is 75miles/hour)

Question #7

Reference Q.581

State whether we can use L'Hopital's rule to solve.

$$\lim_{x \to 1} \frac{3x - 3}{x^4 + x - 2}$$

Question #8

Reference Q.582

State whether we can use L'Hopital's rule to solve.

$$\lim_{x \to 0} \frac{\cos(2x)}{2x}$$

Question #9

Reference Q.583

State whether we can use L'Hopital's rule to solve.

$$\lim_{x \to 0} \frac{5(\sin x - x)}{x^2}$$

Question #10

Reference Q.584

Evaluate:

$$\lim_{x \to 0} \frac{\sin 2x}{e^{2x} - \cos 2x}$$

Question #11

Reference Q.585

Evaluate:

$$\lim_{x \to 0} \frac{\sin 5x}{2x}$$

Question #12

Reference Q.586

Evaluate:

$$\lim_{x\to 0} \frac{\tan 2x}{2x^2}$$

Question #13

Reference Q.587

Evaluate:

$$\lim_{x\to 0} \frac{\sin 5x}{\tan 9x}$$

Question #14

Reference Q.588

Evaluate:

$$\lim_{x\to \infty} \frac{e^x}{\ln x}$$

Question #15

Reference Q.589

Evaluate:

$$\lim_{x\to \infty} x \sin\left(\frac{1}{x}\right)$$

Question #16

Reference Q.590

Evaluate:

$$\lim_{x\to -\infty} (e^{-x} + x)$$

Question #17

Reference Q.9215

Evaluate: $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$

Question #18

Reference Q.9216

Why is L'Hopital's rule not as good a method to evaluate

$$\lim_{x\to \infty} \frac{5x^4 + 3x^3 + 27x}{2x^4 - 33x^2 + 1}$$

as the techniques for evaluating limits of rational functions that go to infinity that you learnt in Chapter 2?

Question #19

Reference Q.9217

The Mean Value Theorem tells us that there is a particular point of interest on the curve $f(x) = e^x$ on the interval $[0, 5]$. What are the coordinates of this point?

Question #20

Reference Q.9218

If $f(x) = \tan(\pi x)$, there is a number c in the interval $[0, 0.25]$ which satisfies the Mean Value Theorem. What is this number?