Calculus

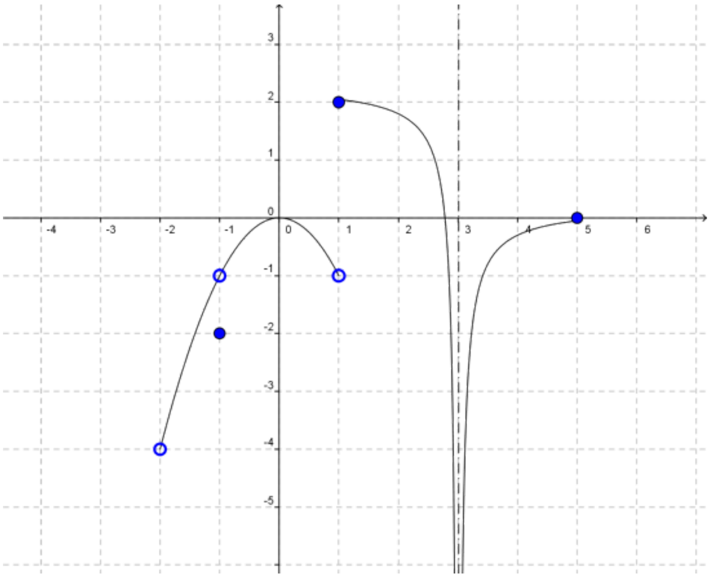
Chapter 2: Limits and Continuity

Lesson 4: Continuity and The Sandwich Theorem

Question #1

Reference Q.208

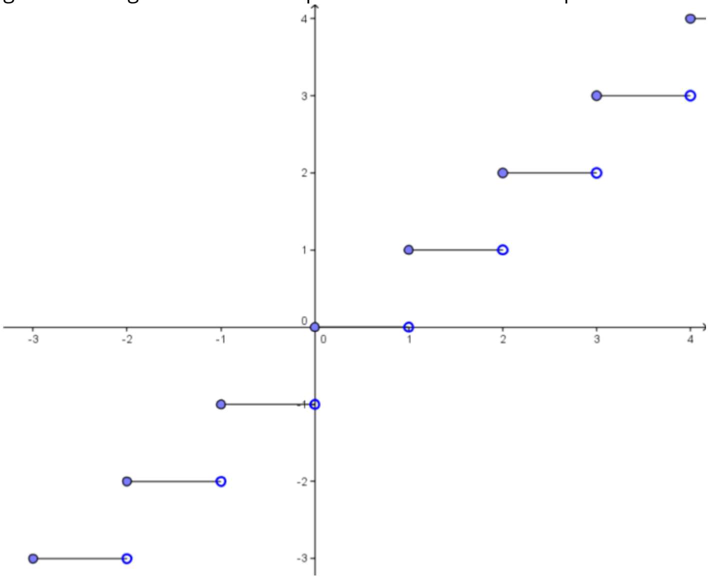
A graph of a function is plotted below.
For which values of x is the function continuous?



Question #2

Reference Q.209

The 'floor function' or 'greatest integer function' in math gives the greatest integer less than or equal to a real number. It is plotted below.



Is the floor function continuous on the following intervals?

a. [1, 3]

b. (1, 3)

c. [1, 2]

d. (1, 2)

e. (1,2]

f. [1,2)

Question #3

Reference Q.210

Sketch a possible graph of a function f that is continuous everywhere except at $x = 1$, and where $\lim_{x \to 1} f(x)$ does not exist.
Possible answer:

Question #4

Reference Q.211

Sketch a possible graph of a function f that is continuous everywhere except at $x = 1$, and where $\lim_{x \to 1} f(x)$ *exists*.
Possible answer:

Question #5

Reference Q.212

If $\lim_{x \to 1} 3f(x)g(x) = 13$, f and g are both continuous functions (meaning they are continuous everywhere), and $f(1) = 1$, find

a. $\lim_{x \to 1} g(x)$

b. b) $g(1)$

Question #6

Reference Q.213

State all the values where $f(x) = x^2 + 2x - 3$ is continuous.

Question #7

Reference Q.214

For which x values is $f(x) = \frac{1-x}{3x^2+6}$ continuous?

Question #8

Reference Q.215

Where is $f(x) = \cos(1 - x^2)$ continuous?

Question #9

Reference Q.216

Where is $f(x) = \frac{x+1}{x^2-1} - \frac{1}{5x}$ continuous?

Question #10

Reference Q.217

Where is $f(x) = \frac{x}{sin3x}$ continuous.

Question #11

Reference Q.218

Where is $f(x) = \frac{x^2+|x|-2}{\sqrt{x}+1}$ continuous?

Question #12

Reference Q.219

Where is $f(x) = \tan x$ continuous?

Question #13

Reference Q.220

Where is $f(x) = \csc^2(x)$ continuous?

Question #14

Reference Q.221

Where is $f(x) = \frac{1}{1-\cos x}$ discontinuous?

Question #15

Reference Q.222

Where is $f(x) = \tan^{-1} x$ continuous?

Question #16

Reference Q.223

Where is $f(x) = \begin{cases} x+3 & x < 3 \\ 3 + \frac{9}{x} & x \geq 3 \end{cases}$ continuous?

Question #17

Reference Q.224

For what value(s) of c will the function

$$f(x) = \begin{cases} 3x^2 + 1, & \text{if } x < 0 \\ \sqrt{4x} + c, & \text{if } x \geq 0 \end{cases}$$

be continuous everywhere?

Question #18

Reference Q.225

For what value(s) of d will the function

$$f(x) = \begin{cases} 10 - 2x^2, & \text{if } x < -3 \\ dx - 2, & \text{if } x \geq -3 \end{cases}$$

be continuous everywhere?

Question #19

Reference Q.226

For what value(s) of k does

$$f(x) = \begin{cases} \sin(kx), & \text{if } x < 0 \\ \cos(x + k), & \text{if} x \geq 0 \end{cases}$$

be continuous everywhere?

Question #20

Reference Q.227

Recall that a "hole" in a function (also called a removable discontinuity) is a point where a limit exists but where the function is discontinuous. Also recall that a hole can be fixed by using a piecewise function and assigning $f(x)$ a value which is the same as the limit, essentially "filling in the hole."

a. a) Verify that $f(x) = \frac{x^2-1}{x+1}$ is not defined at -1, and has a hole, or removable discontinuity there.

b. b) Fix the hole by setting up a piecewise function and assigning $f(x)$ an appropriate value to "fill the hole."

Question #21

Reference Q.228

Determine whether the following function has discontinuities and whether they are removable.

$$f(x) = \frac{x^2 - 3x + 2}{x - 2}$$

Question #22

Reference Q.229

Use a graphing calculator to graph the function $f(x) = \frac{x^2-2x-8}{x-4}$ and see what it looks like. Then, zoom in on the point where $x = 4$. What does the graph tell you about the nature of the discontinuity at $x = 4$? Can you prove it algebraically?

Question #23

Reference Q.230

Determine whether the following function has discontinuities and whether they are removable.

$$f(x) = \left\{ \begin{array}{r} -1, x < -1 \\ 0, -1 \leq x \leq 1 \\ 1, x > 1 \end{array} \right\}$$

Question #24

Reference Q.231

True or False: If f and g are both discontinuous at $x = 0$, then $f + g$ must also *always* be discontinous.

Question #25

Reference Q.232

Prove that the equation $3x^2 - x - 2 = 0$ has at least one solution on the interval $[0, 2]$ by using The Intermediate Value Theorem (IVT).

Question #26

Reference Q.233

Prove that the equation $17x^{14} - 10x = 4$ has at least one solution on the interval $[0, 1]$ by using The Intermediate Value Theorem.

Question #27

Reference Q.234

Without using a graphing calculator, prove that the equation $x - \cos x = 0$ has at least one solution on the interval $\left[0, \frac{\pi}{2}\right]$.

Question #28

Reference Q.235

Evaluate this limit: $\lim_{x \to +\infty} \cos^{-1}\left(\frac{x-3}{x}\right)$

Question #29

Reference Q.236

Evaluate this limit:

$$\lim_{x \to 0^-} \sin(\frac{2}{5x})$$

Question #30

Reference Q.237

Evaluate $\lim_{x \to 0} \dfrac{5}{\tan(x)}$

Question #31

Reference Q.238

Evalute $\lim_{x \to 0} \frac{\sin(2x)}{3x}$

Question #32

Reference Q.239

Evaluate this limit: $\lim_{x \to 0} \frac{\sin(3x)}{\sin(8x)}$

Question #33

Reference Q.240

Evaluate $\displaystyle\lim_{x \to 0} \frac{1 - \cos(x)}{x^3}$.

Question #34

Reference Q.241

Evaluate: $\lim_{x \to 0} \frac{1-\cos x^2}{x}$

Question #35

Reference Q.242

Evalute this limit: $\lim_{x \to 0} \frac{4x^2}{2-2cos^2(x)}$.

Question #36

Reference Q.3211

Draw an example of each of the following discontinuities:

a. an infinite discontinuity

b. a jump discontinuity

c. a removable discontinuity

Question #37

Reference Q.3212

Give an example of a function with: an infinite discontinuity at x = 0, a jump discontinuity at x = 1, and a removable discontinuity at x = -4.

Question #38

Reference Q.9143

Prove that every polynomial, f(x), of order 3 (degree 3) must have at least one real root (i.e. where f(x)=0).

Question #39

Reference Q.17542

Find $\lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

Question #40

Reference Q.17545

Prove $\lim\limits_{x \to 0} x^4 \cos\left(\frac{1}{\sqrt{x}}\right) = 0.$

Question #41

Reference Q.17546

Prove that $\lim\limits_{x \to 0} x^6 3^{\sin\left(\frac{1}{x^2}\right)} = 0.$

Question #42

Reference Q.17547

Prove that $\lim\limits_{x \to 0} x \cos\left(\frac{1}{x}\right) = 0.$

Question #43

Reference Q.17548

It is possible with a graphiing calculator to show that $\cos x \leq \frac{\sin x}{x} \leq 1$ on the interval $[-\pi, \pi]$ by using a graphing calculator. Use this result to prove $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1.$

Question #44

Reference Q.17549

Use the Squeeze Theorem to find

$$\lim_{x \to \infty} \frac{3x^2 + \sin(3x)}{5 - x^2}.$$

Question #45: Intermediate Value Theorem Conditions

Reference Q.64797

One of the conditions of the Intermediate Value Theorem is that the polynomial, $f(x)$, must be continuous over the given interval (a, b). Why is this condition necessary?

Question #46

Reference Q.9135

$f(x)$ is continuous over the interval $(a,b),\ a,b \in \mathbb{R}$. Which of the following are true:

a. $\lim_{x\to t^+} f(x) = f(t)$ for all $t \in (a,b)$.

b. $f(a)$ is finite and exists.

c. For any $p, q \in (a,b)$ such that $f(q) \geq f(p)$ and $f(q) \neq 0$, we can find a real number k such that $f(p) \geq k \cdot f(q)$.

d. There is a number $k \in (a,b)$ such that there is no real number L where $f(k) \leq L$

e. None of the above.

Question #47

Reference Q.9131

If $\lim_{x\to t} f(x) = L$, where $L \in \mathbb{R}$, what of the following are true:

a. $\lim_{x\to t^+} f(x) = L$

b. $f(x)$ is defined at $x = t$.

c. $f(x)$ is continuous at $x = t$

d. $f(t) = L$

e. L is between $f(a)$ and $f(b)$, where $a < t < b$.

f. None of the above.

Question #48

Reference Q.9154

For what values of k is the following function continuous?

$$f(x) = \begin{cases} (x+k)^2, & \text{if } x < 2 \\ x+k, & \text{if } x \geq 2 \end{cases}$$

Question #49

Reference Q.9155

For what values of k is the following function continuous?

$$f(x) = \frac{x(x+1)(x^2-6)}{x^3-kx}$$