

# Pre-Calculus Math 12

## Chapter 2: Function Operations and Conics

### Lesson 1: Basic Function Operations

#### Question #1

Reference Q.11164

Consider the functions  $f(x) = 3x + 7$  and  $g(x) = 2x - 4$ , defined for all real numbers.

a. Determine the following functions in simplest form.

ii.  $(f + g)(x)$

iii.  $(f - g)(x)$

iv.  $(g - f)(x)$

b. Evaluate

iii.  $(f + g)(5)$

iv.  $(f - g)(-3)$

v.  $(g - f)(0)$

#### Question #2

Reference Q.11165

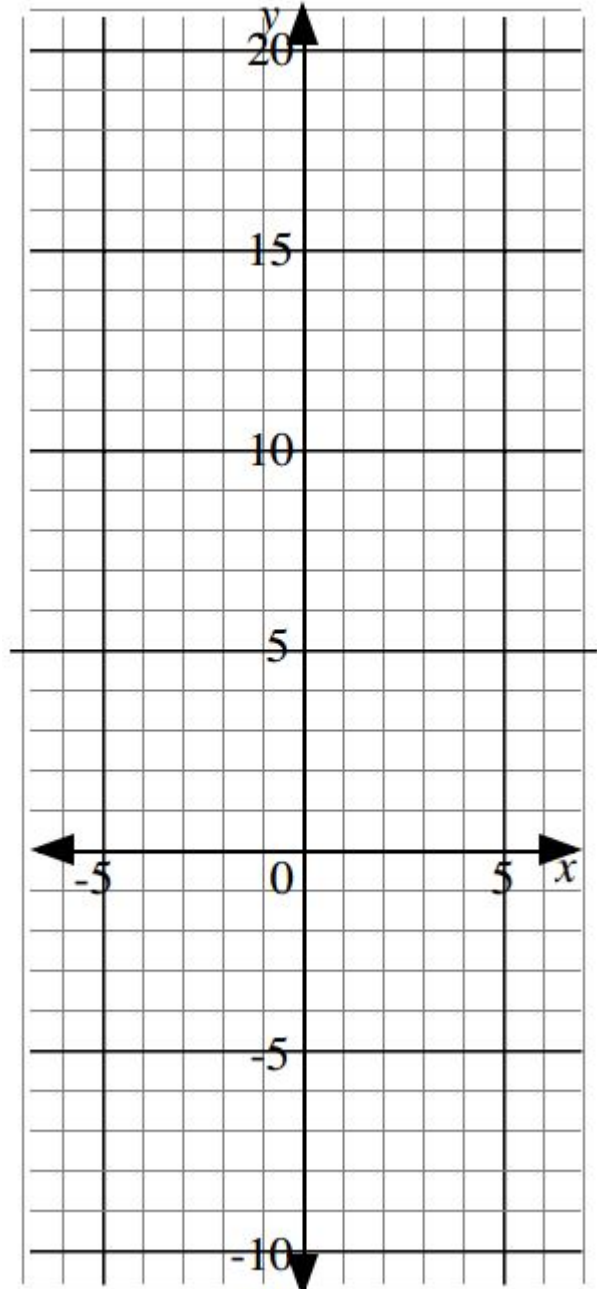
If  $P(x) = x^2 - 7x + 3$  and  $Q(x) = 2x + 9$ , evaluate  $(P - Q)(-3)$  in two different ways.

🔗 **Question #3**

Reference Q.11166

Two functions  $f(x)$  and  $g(x)$  are defined for all real numbers. The table of values below shows the values of  $f$  and  $g$  for certain values of  $x$ .

| $x$ | $f(x)$ | $g(x)$ | $(f + g)(x)$ | $(f - g)(x)$ |
|-----|--------|--------|--------------|--------------|
| -4  | 9      | -8     |              |              |
| -3  | 4      | -7     |              |              |
| -2  | 1      | -6     |              |              |
| -1  | 0      | -5     |              |              |
| 0   | 1      | -4     |              |              |
| 1   | 4      | -3     |              |              |
| 2   | 9      | -2     |              |              |
| 3   | 16     | -1     |              |              |
| 4   | 25     | 0      |              |              |

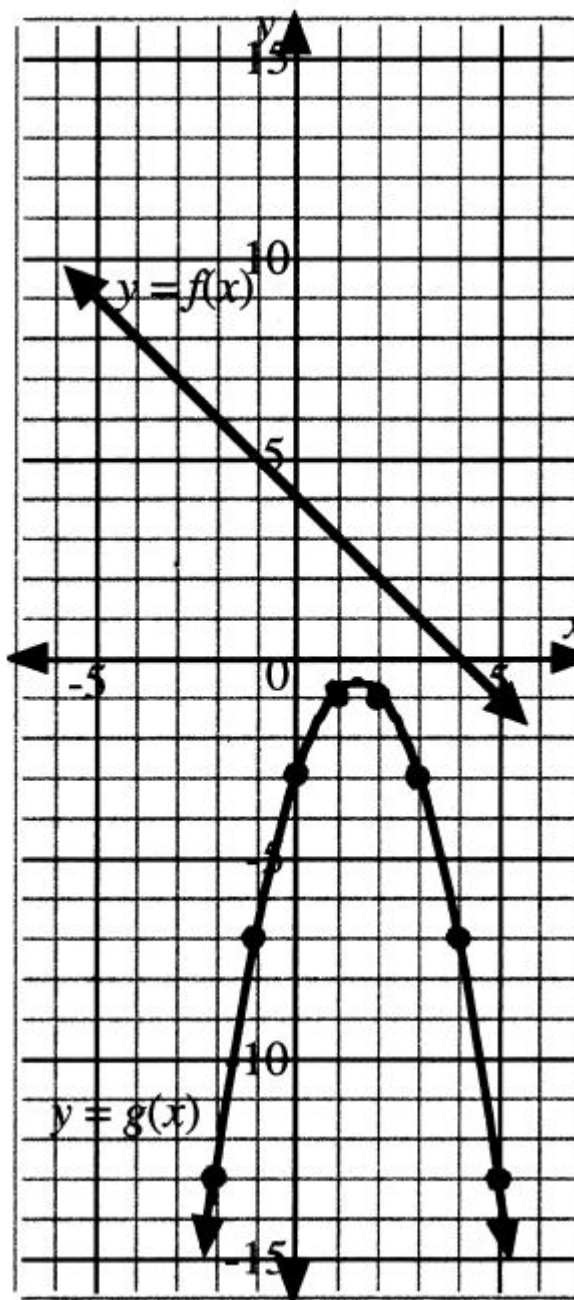


- Plot the points  $(x, f(x))$  on the grid and sketch the graph of  $y = f(x)$  for  $x \in \mathbb{R}$ .
- Plot the points  $(x, g(x))$  on the grid and sketch the graph of  $y = g(x)$  for  $x \in \mathbb{R}$ .
- Complete the tables above for  $(f + g)(x)$  and  $(f - g)(x)$  and sketch the graphs of  $y = (f + g)(x)$  and  $y = (f - g)(x)$  for  $x \in \mathbb{R}$ .
- The functions  $f$  and  $g$  on the previous page are  $f(x) = x^2 + 2x + 1$  and  $g(x) = x - 4$ . Write expressions for the functions  $(f + g)(x)$  and  $(f - g)(x)$ .
- Use a graphing calculator to graph the functions  $y = (f + g)(x)$  and  $y = (f - g)(x)$  from part d) and compare these graphs with the graphs from part c).
- State the domains of the functions  $y = (f + g)(x)$  and  $y = (f - g)(x)$ .
- Determine the values of  $(f + g)(8)$  and  $(f - g)(12)$ .
- In this example, the sum and difference of a linear function and a quadratic function are quadratic functions. Can you find a linear function and a quadratic function whose sum or difference is not a quadratic function?

### Question #4

Reference Q.11167

The graph of two quadratic functions,  $y = f(x)$  and  $y = g(x)$ ,  $x \in \mathbb{R}$  are shown on the grid.



- Use these graphs to sketch the graphs of  $y = (f + g)(x)$  and  $y = (f - g)(x)$  for  $x \in \mathbb{R}$ .
- The functions  $f$  and  $g$  above are  $f(x) = 4 - x$  and  $g(x) = -x^2 + 3x - 3$ . Write expressions for the functions  $(f + g)(x)$  and  $(f - g)(x)$ .
- Use a graphing calculator to graph the functions  $y = (f + g)(x)$  and  $y = (f - g)(x)$  from part b) and compare these graphs with the graphs from part a).
- State the domains of the functions  $(f + g)(x)$  and  $(f - g)(x)$ .

e. Evaluate

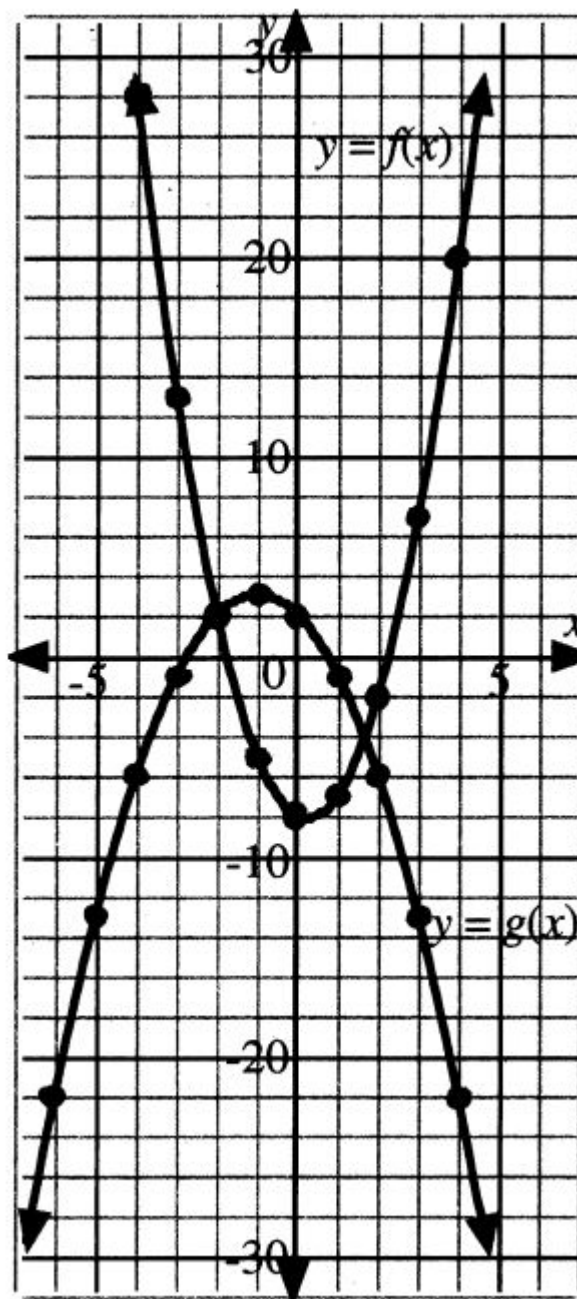
i)  $(f + g)(-10)$

ii)  $(f - g)(\sqrt{3})$

### Question #5

Reference Q.11168

The graph of two quadratic functions,  $y = f(x)$  and  $y = g(x)$ ,  $x \in \mathbb{R}$  are shown on the grid.



- Use these graphs to sketch the graphs of  $y = (f - g)(x)$  and  $y = (g - f)(x)$  for  $x \in \mathbb{R}$ .
- How do the graphs of  $y = (f - g)(x)$  and  $y = (g - f)(x)$  relate to each other?
- The functions  $f$  and  $g$  above are  $f(x) = 2x^2 - x - 8$  and  $g(x) = -x^2 - 2x + 2$ . Write expressions for the functions  $(f - g)(x)$  and  $(g - f)(x)$ .
- Use a graphing calculator to graph the functions  $y = (f - g)(x)$  and  $y = (g - f)(x)$  from part c) and compare these graphs with the graphs from part a).

e. Evaluate

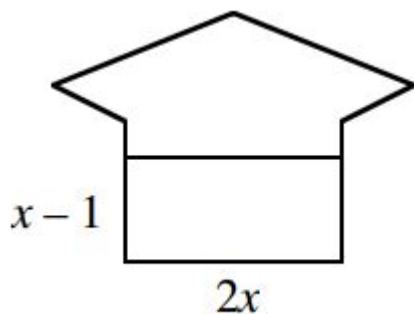
i)  $(f - g)\left(\frac{1}{2}\right)$

ii)  $(g - f)(2\sqrt{2})$

### Question #6

Reference Q.11169

The figure shown has an area  $A(x) = 2x^2 + 8x - 3 \text{ cm}^2$ .



- Write an expression for the area,  $B(x)$ , of the bottom rectangular part of the figure.
- Find an expression in simplest form for the area,  $T(x)$ , of the top part of the figure.
- If the area of the top is  $9 \text{ cm}^2$ , determine the value of  $x$ .

### Question #7

Reference Q.11171

The graph of  $y = f(x)$  includes the points  $(2, 5)$ ,  $(4, 15)$ ,  $(7, 30)$ .

The graph of  $y = g(x)$  includes the points  $(2, 6)$ ,  $(3, 16)$ ,  $(4, 30)$ .

If  $f$  and  $g$  are defined for all real values of  $x$ , then which of the following points **must** lie on the graph of  $y = (f - g)(x)$ ?

- $(0, -1)$
- $(0, 1)$
- $(2, -1)$
- $(3, 30)$

### Question #8

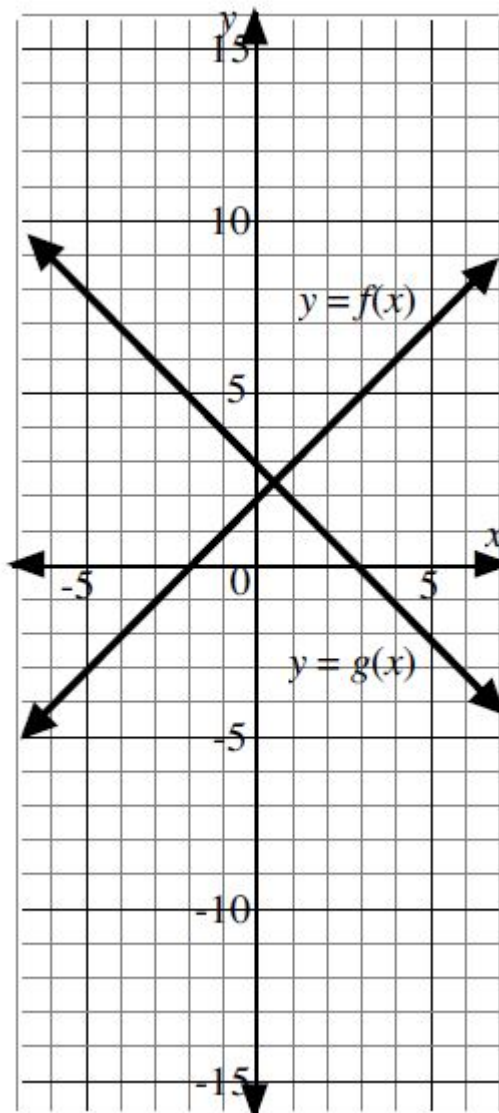
Reference Q.11172

If  $f(x) = \sqrt{3x} - 2$  and  $g(x) = 5\sqrt{3x}$ , then  $2(g - f)(3)$ , to the nearest tenth, is \_\_\_\_\_.

### Question #9

Reference Q.11173

Consider functions  $f$  and  $g$  defined for all real numbers. Partial graphs of the functions are shown on the grid. Both functions have integer values when  $x$  is an integer.



a. Complete the table below for  $(fg)(x)$ .

| $x$ | $f(x)$ | $g(x)$ | $(fg)(x)$ |
|-----|--------|--------|-----------|
| -4  |        |        |           |
| -3  |        |        |           |
| -2  |        |        |           |
| -1  |        |        |           |
| 0   |        |        |           |
| 1   |        |        |           |
| 2   |        |        |           |
| 3   |        |        |           |
| 4   |        |        |           |
| 5   |        |        |           |

b. Plot the points from the table which will fit on the grid and complete the sketch of  $y = (fg)(x)$  for  $x \in \mathbb{R}$ .

c. The functions  $f$  and  $g$  above are  $f(x) = 2 + x$  and  $g(x) = 3 - x$ .

Write a simplified expression for the function  $(fg)(x)$ .

d. Use a graphing calculator to graph the function  $y = (fg)(x)$  from part c) and compare this graph with the graph from part b)

e. Determine the domain and range of the function  $(fg)(x)$ .

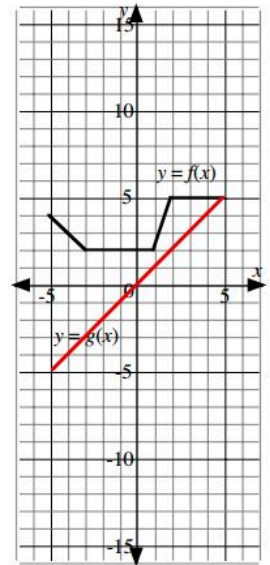
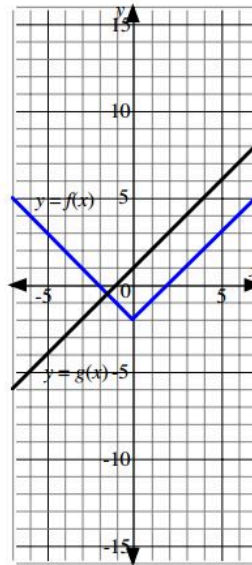
f. Calculate the value of  $(fg)\left(-\frac{1}{2}\right)$  in two different ways.

g. How does the graph of  $(gf)(x)$  compare to the graph of  $(fg)(x)$ ?

## Question #10

Reference Q.11220

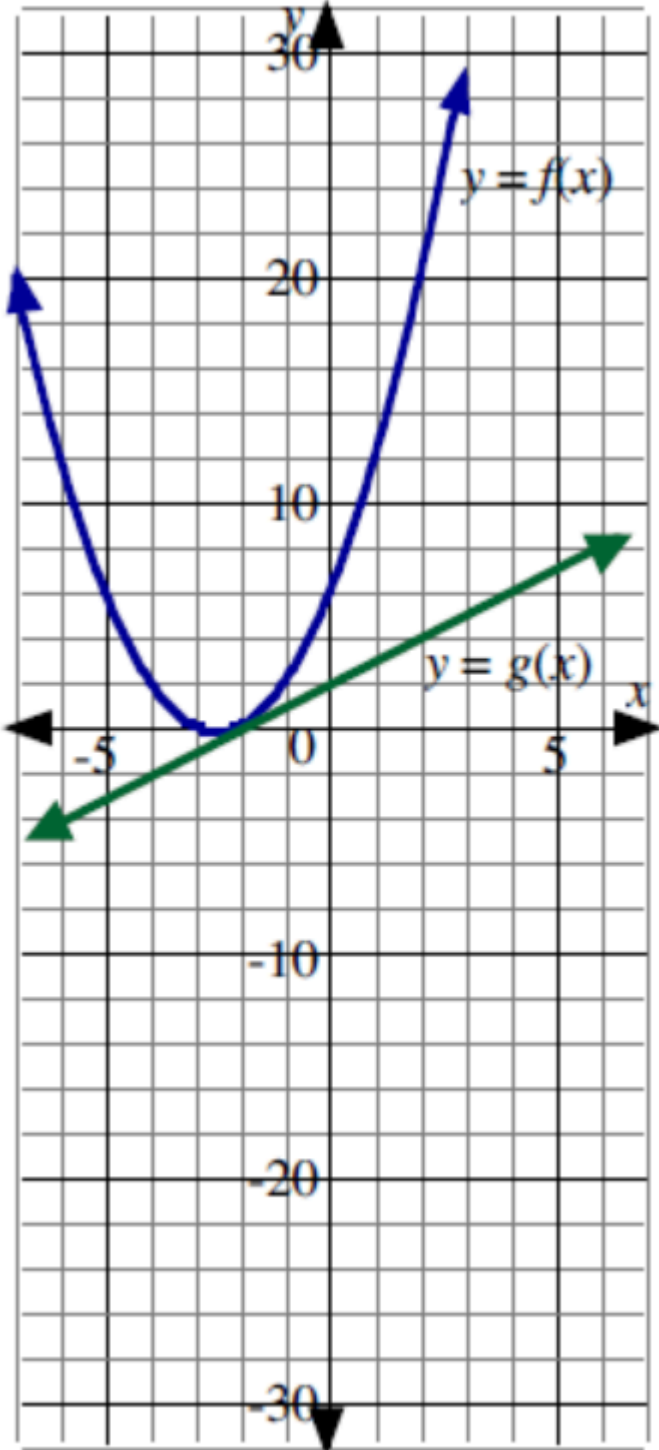
Partial graphs of functions  $f$  and  $g$  are shown on the grids. In each case, sketch the partial graph of the function that is the product of these two functions.



## Question #11

Reference Q.11221

Consider functions  $f$  and  $g$  defined for all real numbers. Partial graphs of the functions are shown on the grid.



a. Complete the table using the values from the grid.

| $x$ | $f(x)$ | $g(x)$ | $(f + g)(x)$ | $(fg)(x)$ |
|-----|--------|--------|--------------|-----------|
| -6  |        |        |              |           |
| -5  |        |        |              |           |
| -4  |        |        |              |           |
| -3  |        |        |              |           |
| -2  |        |        |              |           |
| -1  |        |        |              |           |
| 0   |        |        |              |           |
| 1   |        |        |              |           |
| 2   |        |        |              |           |

b. Plot the points from the table which will fit on the grid and sketch the graphs of  $y = (f + g)(x)$  and  $y = (fg)(x)$ .

c. The functions  $f$  and  $g$  above are  $f(x) = (x + 3)(x + 2)$  and  $g(x) = x + 2$ .

iv. Write an expression for the function  $(f + g)(x)$  in expanded form.

v. Write an expression for the function  $(fg)(x)$  in expanded form.

d. Use a graphing calculator to graph the function from part c) and verify the sketches from part b).

e. Complete the statement.

"In this question the product of a linear function and a quadratic function is a \_\_\_\_\_ function."

f. Can you find two functions, one whose graph is a straight line, and one whose graph is a parabola, such that the graph of the product of the two functions is not the graph of a cubic function?

## Question #12

Reference Q.11222

Consider the functions  $f(x) = 2\sqrt{x} - 5$  and  $g(x) = \sqrt{x} - 5$ .

a. Determine the following functions in simplest form and state the domain of each function.

ii.  $(f + g)(x)$

iii.  $(g - f)(x)$

iv.  $(gf)(x)$

b. Evaluate.

iii.  $(g + f)(4)$

iv.  $(f - g)(49)$

v.  $(fg)(0.25)$

⊙ **Question #13**

Reference Q.11223

The points  $(4, 1)$ ,  $(7, 4)$ ,  $(9, 6)$  lie on the graph of

$$y = P(x), x \in \mathbb{R}.$$

The points  $(4, 3)$ ,  $(7, 6)$ ,  $(9, 8)$  lie on the graph of

$$y = Q(x), x \in \mathbb{R}.$$

Which of the following points must lie on the graph of

$$y = (PQ)(x)?$$

- A.  $(4, 3)$
- B.  $(16, 3)$
- C.  $(49, 24)$
- D.  $(63, 6)$

⊙ **Question #14**

Reference Q.11224

Consider the function  $f(x) = 6 - 3x$  and  $g(x) = 5 - 2x$ . If the

function  $(fg)(x)$  is written in the form  $ax^2 + bx + c$ , then the value of  $a - b + c$  is \_\_\_\_\_.