Pre-Calculus Math 12

Chapter 5: Polynomial and Rational Functions

Lesson 1: Dividing Polynomials

Question #1

Reference Q.11874

Which of the following are not polynomial functions of x? Explain.

a. $f(x) = \frac{1}{2}x^2 + 5x$

b. $f(x) = x^{-4} + x^3$

c. $f(x) = \sqrt{x} + 3x - 1$

d. $f(x) = 3^x + 1$

e. $f(x) = 4(\sqrt{x} - 1)(\sqrt{x} + 1)$

f. $f(x) = \sqrt{10x^4} + 3x$

g. $f(x) = \sqrt{x^2 + 4}$

Question #2

Reference Q.11878

State the degree, leading coefficient and constant term for each of the following polynomial functions.

a. $P(x) = 5x^3 - 7x^4 + 2$

b. $P(x) = 4x^2 - x^4 + 2x^3 + x^4$

c. $P(x) = 4(4x + 1)^2(2x^2 - x + 5)$

d. $P(x) = 3(4x + 1)^2 - (4x^2 + x - 1)$

e. $P(x) = (x + 1)(x + 2)(x - 3)$

f. $g(x) = -5(2x^3 + 3x - 2)^4$

Question #3

Reference Q.11882

Which of the following is a polynomial of degree 5?

a) $5x^4 + 5x$

b) $\frac{3}{x^5} + 5$

c) $x^2 - 3x^5 + x - 4$

d) $x^5 - 2x^7$

e) $2x^5 + 3x - 7x^{-3}$

f) $(x^2 + 3)(x^3 + 4)$

Question #4

Reference Q.11886

For each polynomial, use substitution to find the indicated value.

a. $P(x) = 3x^3 - 5x^2 - 2; P(-3)$

b. $P(x) = 4x^4 - x^3 - x + 3;\ P\left(\frac{1}{2}\right)$

Question #5

Reference Q.11887

Some of the following functions are polynomial functions when simplified. For those which are, answer the following.
i) Write the expression in the simplest polynomial form in descending powers of x.
ii) Name the type of polynomial (integral, rational, real) according to its coefficients.
iii) State the degree, leading coefficient, and constant term.

a. $f(x) = 3x^{-4} + 2x^3 - 3x^{-4} + 2x^3$

b. $g(x) = (\sqrt{3}x - \sqrt{2})(\sqrt{3}x + \sqrt{2})$

c. $h(x) = 8x^5 - \sqrt{36x} + 2$

d. $f(x) = 3 - \sqrt{10x^2} + 3x^4$

e. $g(x) = \frac{7x^3 - 3x^2 + 2x}{2}$

f. $g(x) = 2x^2 - 3x^{-1} + 5x^2 + 7x^{-1}$

Question #6

Reference Q.11889

Determine the values of a and b in $P(x) = ax^3 + bx^2 + 3x - 4$ if $P(1) = -2$ and $P(2) = 2$.

Question #7

Reference Q.11890

Determine the values of a, b and c in $P(x) = ax^2 + bx + c$ if $P(0) = 1, P(1) = 6, P(-1) = 2$.

Question #8

Reference Q.12754

Consider the polynomial function $P(x) = px^3 + qx + r$, where $P(0) = 1, P(1) = 3,$ and $P(-2) = -33.$ The value of p, to the nearest tenth, is ____.

Question #9

Reference Q.12755

The degree of the polynomial $4(x+1)^3 + (x^3 - 2)^2 - x^2(x^4 + 12)$ is ____.

Question #10

Reference Q.11920

Divide the following polynomials. Express the answer in the form $P = DQ + R.$

a. $\dfrac{x^2 + 5x + 4}{x + 2}$

b. $\dfrac{2x^2 - 5x + 2}{x - 3}$

c. $\dfrac{6x^2 - 5x - 3}{x - 1}$

Question #11

Reference Q.11921

Divide the following polynomials. Express the answer in the form $\dfrac{P}{D} = Q + \dfrac{R}{D}.$

a. $\dfrac{a^3 - a^2 - 4a + 12}{a - 2}$

b. $\dfrac{3x^3 - x^2 + 2x + 4}{x + 4}$

Question #12

Reference Q.11922

Determine the quotient and remainder when the following polynomials are divided.

a. $\dfrac{6x^2 - 5x + 7}{2x - 3}$

b. $\dfrac{9x^2 - 9}{3x + 1}$

c. $\dfrac{12x^3 - 5x^2 + x}{4x - 3}$

Question #13

Reference Q.11923

a. Explain how to determine if $x + 5$ is a factor of $x^3 + 125.$

b. Use the process in a) to determine if $x + 5$ is a factor of $x^3 + 125.$

Question #14

Reference Q.11924

Given that the degree of $D(x)$ is 4, state the possible degrees of $R(x)$ in $P(x) = D(x)Q(x) + R(x).$

Question #15

Reference Q.11925

A rectangular carpet has an area of $x^3 - 5x + 12$ square meters.

a. If the width of the carpet is $x + 3$ meters, determine an expression for the length of the carpet.

b. If the width of the carpet is 8 meters, determine the length of the carpet.

Question #16

Reference Q.11926

The division shows a polynomial expression in x, written as $P(x)$, being divided by a binomial.

$$3x-2\overline{)\,P(x)}\quad x^2-3x+2$$

$$\overline{17}$$

a. Write $P(x)$ in the form $D(x)Q(x)+R(x)$

b. Write $P(x)$ in the form ax^3+bx^2+cx+d.

Question #17

Reference Q.11927

When a third degree polynomial is divided by $x+5$, the quotient is x^2-2x-1 and the remainder is 7. Express the polynomial in the form ax^3+bx^2+cx+d.

Question #18

Reference Q.11928

When $(3z^4+6z^3-18z)$ is divided by $(z+3)$, the remainder is

A. -45
B. 45
C. -135
D. 135

Question #19

Reference Q.11929

A rectangle has an area $8x^2-14x-15\ \text{cm}^2$ and a length of $4x+3\ \text{cm}$. The perimeter of the rectangle can be written in the form $ax+b\ \text{cm}$. The value of $a+b$ is _____.

Question #20

Reference Q.11930

When the polynomial ax^3+bx^2+cx+d is divided by $3x-2$, the quotient is $2x^2+2x+3$ and the remainder is 7. Determine the values of a, b, c, and d.

Question #21

Reference Q.11931

In the partially completed synthetic division below, a polynomial $P(x)$ is divided by $x-2$.

1	-2	6	3
1	0	6	15

a. State the polynomial $P(x)$.

b. State the quotient.

c. State the remainder.

d. Write the above synthetic division in the form of the division algorithm.

Question #22

Reference Q.11932

Use synthetic division to divide the polynomial by the binomial and express each in the form $P(x)=D(x)Q(x)+R(x)$.

a. $x^3+2x^2+3x+6\div x-2$

b. $2x^3-4x^2-5x+9\div x+2$

c. $x^4-x^2+7\div x+1$

d. $2y^4-y^5-y^3+4y\div y-3$

Question #23

Reference Q.11933

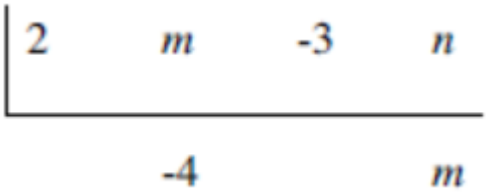
Determine p, q, and r in the partially completed synthetic divisiom below in which the divisor is $x-1$.

2	3	q	1
2	p	7	r

Question #24

Reference Q.11934

Determine m and n in the partially completed synthetic division below in which the divisor is $x + 2$.



Question #25

Reference Q.11935

Find the remainder on dividing $x^3 - 3x^2 + x + 8$ by $x - 2$.
Compare this with $f(2)$ where $f(x) = x^3 - 3x^2 + x + 8$.

Question #26

Reference Q.11936

Find the remainder on dividing $12 - 5x + 3x^2 + 2x^3$ by $x + 3$.
Compare this with $P(-3)$ where
$P(x) = 12 - 5x + 3x^2 + 2x^3$.

Question #27

Reference Q.11937

When $2x^3 + ax^2 - 3x + 4$ is divided by $x + 1$, the remainder is 8. Determine the value of a.

Question #28

Reference Q.11938

Divide $9x^3 + 18x^2 - 13x + 5$ by $3x - 1$ using synthetic division and write the division in the form $P = DQ + R$.

Question #29

Reference Q.11939

Divide $4x^3 + 11x^2 - 14x - 9$ by $4x + 3$ using synthetic division and write the division in the form $P = DQ + R$.

Question #30

Reference Q.11940

When the polynomial $2a^3 - 7a + 6$ is divided by $a - 4$, the remainder is

A. -94
B. 10
C. 66
D. 106

Question #31

Reference Q.11941

When the polynomial $3y^3 - 4y^2 + by + 6$ is divided by $y + 2$, the remainder is -40. The value of b is ___.