AP Calculus AB/BC - Limits and Continuity

Show all your work. Clearly indicate the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations. Use complete sentences for all explanations.

1. Prove that if $f(x)$ is continuous on an interval, then so is $|f(x)|$.

2. Compute the following limits. If a limit does not exist, explain why.

   a) $\lim_{x\to 5} \frac{2x^2-9x-5}{x-5}$

   b) $\lim_{x\to -2} \frac{\sqrt{x+27}-5}{x+2}$

   c) $\lim_{x\to \frac{1}{3}} \frac{3x^2+5x-2}{|3x-1|}$

d) $\lim_{x\to\frac{\pi}{4}} \frac{\cos\theta-\sin\theta}{(\cos\theta)(1-\tan^4\theta)}$

e) $\lim_{x\to-\infty} \frac{\sqrt[3]{16x^3-90}}{3-x}$

f) $\lim_{x\to\infty} \frac{4e^{5x}+2e^x+x^3}{3e^{3x}-12e^{5x}}$

3. Use the squeeze theorem to find $\lim_{x\to0} x^4 \sin\left(\frac{3}{x}\right)$

4. Find the points of discontinuities of the function (if any) and describe their types.

$$f(x) = \frac{x^4 - 4x^3 - 5x^2}{x^2 + x}$$

5. We have a function $f: [-5,5] \to R$. We are told that it has two zeros on its domain and the following (incomplete) list of points.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	4	1	-4	1	5	-3	-1	2	3	-2	-3

We use this information to conclude that $f(x)$ is not continuous. Write a paragraph explaining the reasoning.

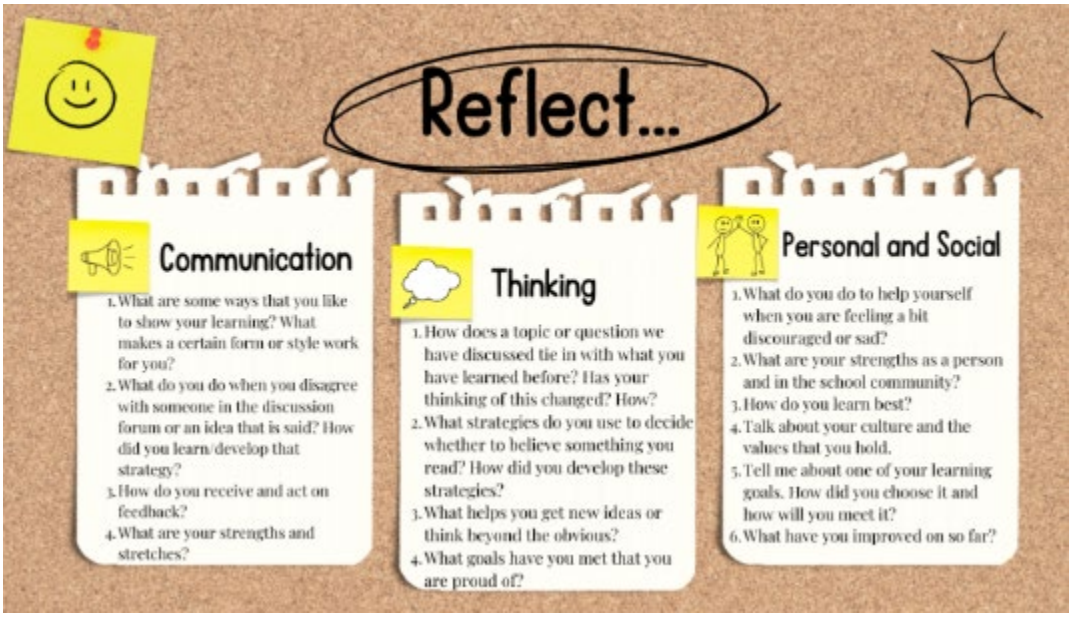
6. Find values of constants A and B such that

$$\lim_{x\to 0}\frac{\sqrt{A+Bx}-\sqrt{3}}{x}=\sqrt{3}$$

7. Evaluate this limit without using l'Hopital's Rule.

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

8. Think about your learning journey in this unit. Pick a category and write a response to ONE of the category's questions in a few complete sentences.



~The End~

Quick checklist:

Did you…

- O State your final answer and explanations using full sentences?
- O Explain in a few sentences every time you have a Yes/No answer?
- O Use equal signs correctly where necessary?