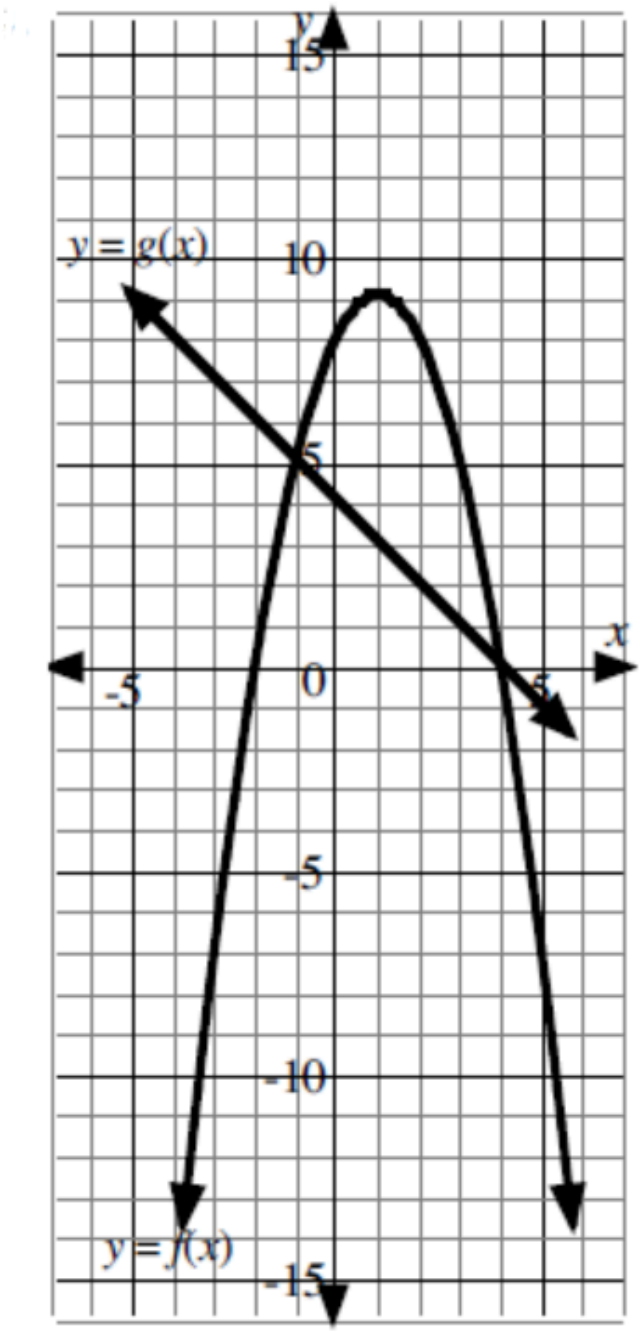
Lesson 2: Composition of Functions

Question #1

Reference Q.11225
Two functions f(x) and g(x) are defined for all real numbers. The graphs of the functions are shown on the grid.



a. Complete the table below.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
–4	–16	8	–16/8 = –2
–3			
–2			
–1			
0			
1			
2			
3			
4			
5			

b. Expain why the domain of the function $\left(\frac{f}{g}\right)(x)$ is not $x \in \mathbb{R}$.

c. Sketch the graph of $y = \left(\frac{f}{g}\right)(x)$ showing the domain restriction by drawing an open circle on the graph.

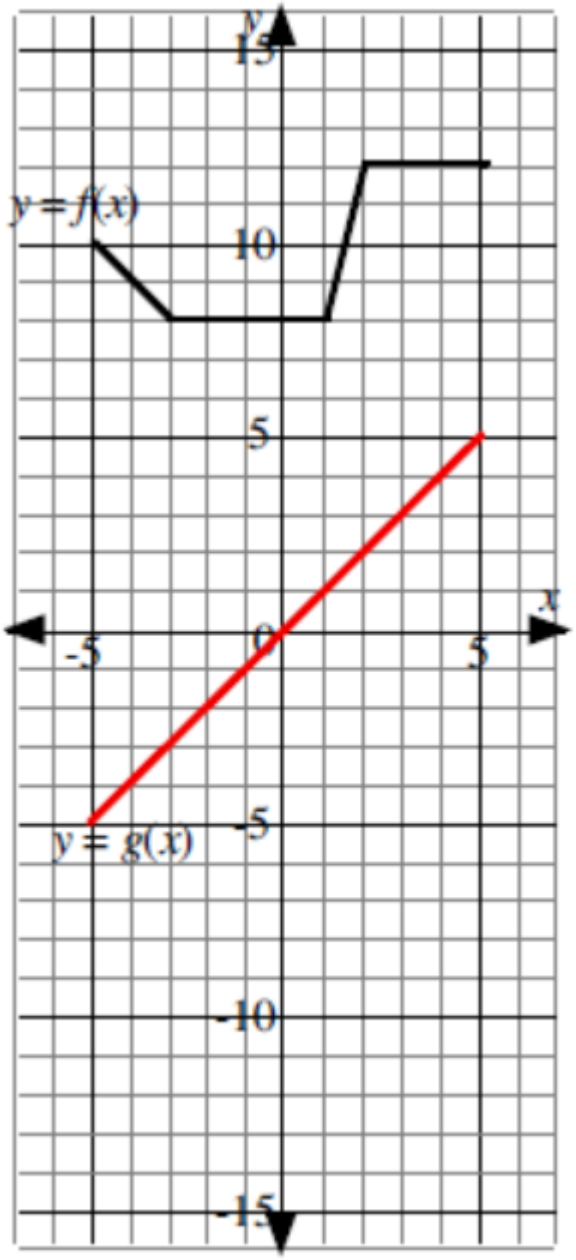
d. The functions f and g abover are $f(x) = 8 + 2x - x^2$ and $g(x) = 4 - x$. Write and simplify an expression for the function $\left(\frac{f}{g}\right)(x)$, including the domain restriction.

e. Evaluate $\left(\frac{f}{g}\right)(12)$.

Question #2

Reference Q.11226

Two functions f(x) and g(x) are defined for all real numbers. The graphs of the functions are shown on the grid.



a. Complete the table below. Plot the points of $y = \left(\frac{f}{g}\right)(x)$, but do not connect the points at this time.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
–5			
–4			
–3			
–2			
–1			
0			
1			
2			
3			
4			
5			

b. State the domain of the function $\left(\frac{f}{g}\right)(x)$.

c. To investigate the behaviour of the function $\left(\frac{f}{g}\right)(x)$ near the domain restriction, complete the table below and plot the points on the grid.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
–0.5			
–0.25			
0.25			
0.5			

d. Connect all the points on the grid and complete the graph of $y = \left(\frac{f}{g}\right)(x)$.

Question #3

Reference Q.11227

In each case, write and simplify (where possible) an expression for $\left(\frac{f}{g}\right)(x)$. Include an domain restrictions.

a. $f(x) = x + 6, g(x) = x + 3$

b. $f(x) = x^2 - x - 20, g(x) = x - 5$

c. $f(x) = x + 4, g(x) = x^2 + x - 12$

d. $f(x) = 2x^2 - 7x - 15, g(x) = x - 5$

Question #4

Reference Q.11228

Given $f(x) = 5x - 10$ and $g(x) = x - 2$, determine the following in simplest form and state any restrictions on x.

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

Question #5

Reference Q.11229

Given $f(x) = x^2 - 9$ and $g(x) = x + 3$, determine the following functions in simplest form and state the restrictions on the variable.

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

Question #6

Reference Q.11230

Consider the functions $f(x) = 6x^2 + 5x - 6$, and $g(x) = 6x^2 - 13x + 6$

a. State the domains of f and g.

b. Write an expression in simplest form for $\left(\frac{f}{g}\right)(x)$. State the domain.

c. **Show** two different ways to evaluate $\left(\frac{f}{g}\right)(4)$.

Question #7

Reference Q.11231

In each case, find two functions f and g such that

a. $(f + g)(x) = x^2 + 3x$

b. $(f - g)(x) = x^2 + 3x$

c. $(fg)(x) = x^2 + 3x$

d. $\left(\frac{f}{g}\right)(x) = x^2 + 3x$

Question #8

Reference Q.11232

Given $f(x) = \frac{x}{x - 3}$ and $g(x) = \frac{2x}{x + 1}$, determine the following functions in simplest form and state any restrictions on x.

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

Question #9

Reference Q.11233

Given $f(x) = x + 1$ and $g(x) = x^2 - 1$, determine the following functions in simplest form and state any restrictions on x.

a. $3(f - g)(x)$

b. $(f + f)(x) - g(x)$

c. $\left(\frac{g}{f}\right)(x)$

Question #10

Reference Q.11234

Consider the functions $f(x) = \frac{x-4}{x+2}$ and $g(x) = \frac{x-3}{x-1}$. Which of the following are restrictions for $\left(\frac{f}{g}\right)(x)$?

A. –2 and 1 only
B. –2, 1, and 3 only
C. –2, 1, and 4 only
D. –2, 1, 3, and 4

Question #11

Reference Q.11235

If $f(x) = 2x^2$ and $g(x) = \frac{x-4}{2x}$, determine the values of

1. $(fg)(3)$
2. $(f - g)(4)$
3. $\left(\frac{f}{g}\right)(2)$
4. $(f + g)(1)$

Rearrange the four answers in increasing order. Write the question number corresponding to the smallest answer as first digit, the question number corresponding to the second smallest answer as second digit, etc.

Question #12

Reference Q.11236

[Context - The consumption of non-renewable resources has been of concern to aboriginals long before it has become a global concern. A modern example is petroleum consumption, of which cars are a major contributor.] The 305 HP V6 2012 Mustang Sports Car has a highway fuel efficiency of approximately 3.2 gallons per 100 miles. The cost of gasoline is $3.60 per gallon.

a. The volume, v gallons, of fuel used can be written as a function of the distance, d miles, travelled. Complete the following for v in terms of d.
$v = f(d) =$ _____

b. The cost, C dollars, of gasoline used can be written as a function of v. Complete the following for C in terms of v.
$C = g(v) =$ _____

c. We can find the cost of gasoline in terms of the distance tranvelled by combining these two functions. If we substitute the formula for the first function into the formula for the second function, we can write C as a function of d. Complete the following:
$C = h(d) =$

Question #13

Reference Q.11237

The function $p(d) = 0.62d$ converts Canadian dollars into British pounds.
The function $e(p) = 1.15p$ converts British pounds into euros.

a. Determine the function $e(d)$ that converts Canadian dollars into euros.

b. Use these functions to convert C$2000 into euros.

c. Determine the function $d(e)$ that converts euros into Canadian dollars and convert 2000 euros into Canadian dollars (to the nearest cent).

Question #14

Reference Q.11239

A composite function h(x) is given. In each case, complete the diagram and write h(x) as a composition of two functions, f ang g, where $h(x) = f(g(x))$.

a. $h(x) = \sqrt{x} - 4$

$g(x) =$ _____ $f($ ___ $) =$ _____ $f(x) =$ _____

b. $h(x) = \frac{1}{x+3}$

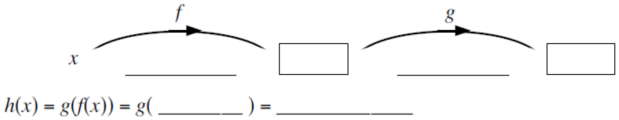
$g(x) =$ _____ $f($ ___ $) =$ _____ $f(x) =$ _____

Question #15

Reference Q.11240

Consider two functions $f(x) = x + 2$ and $g(x) = x^2$.

a. Complete the diagram to determine a formula for the composite function $h(x) = g(f(x))$.



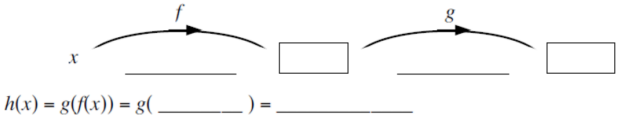
b. Use a similar technique to determine a formula for the composite function $k(x) = f(g(x))$.

Question #16

Reference Q.11241

Consider two functions $f(x) = \sqrt{x}$ and $g(x) = 4x$.

a. Complete the diagram to determine a formula for the composite function $h(x) = g(f(x))$.



b. Use a similar technique to determine a formula for the composite function $k(x) = f(g(x))$.

Question #17

Reference Q.11242

For each pair of functions, write a formula for $(f(g(x))$.

a. $f(x) = 2x + 1, g(x) = 5x$

b. $f(x) = 5x - 2, g(x) = x^3$

c. $f(x) = 2^x, g(x) = x + 4$

Question #18

Reference Q.11243

For each pair of functions, write a formula for $g(f(x))$.

a. $f(x) = 2 - x, g(x) = |x + 2|$

b. $f(x) = 2x + 1, g(x) = x^4$

c. $f(x) = 3^x, g(x) = x - 1$

Question #19

Reference Q.11244

If $f(x) = x + 4$ and $g(x) = x - 1$, determine the value(s) of x for which

a. $f(g(x)) = 50$

b. $(fg)(x) = 50$

Question #20

Reference Q.11245

Consider the functions $f(x) = \sqrt{x - 3}$ and $g(x) = x^2 + 2$.

a. Find expressions for $f(g(x))$ and $g(f(x))$.

b. Determine the domains of f, g, $f(g(x))$, and $g(f(x))$.

c. Determine the ranges of f, g, $f(g(x))$, and $g(f(x))$.

d. In each case, sketch he graph of the functions indicated.

v. $y = f(x)$
vi. $y = g(x)$
vii. $y = f(g(x))$
viii. $y = g(f(x))$

Question #21

Reference Q.11246

If $f(x) = 2x + 3$ and $g(x) = 5 - 2x$, determine the value of:

a. $f(g(5))$

b. $g(f(-3))$

c. $(f \circ g)(0)$

d. $-2(g \circ f)(0)$

Question #22

Reference Q.11247

If $f(x) = 2\sqrt{x}$ and $g(x) = 2 + 2x$, determine the value of:

a. $f(g(7))$

b. $g\left(f\left(\frac{1}{4}\right)\right)$

c. $(f \circ g)(5)$

d. $3(g \circ f)(5)$

Question #23

Reference Q.11248

Find $(f \circ g)(x), (g \circ f)(x),$ and $(f \circ f)(x)$ for the following. State any domain restrictions.

a. $f(x) = -2x, g(x) = x^2 - 3$

b. $f(x) = \dfrac{1}{3 - x}, g(x) = x^2$

c. $f(x) = 3x, g(x) = \sqrt{x - 2}$

Question #24

Reference Q.11249

Given $f(x) = 4 - x$ and $g(x) = 3\sqrt{5x},$ then $(f \circ g)(5)$ is equal to

A. -71
B. -11
C. -1
D. 35

Question #25

Reference Q.11250

Given $f(x) = \dfrac{1}{x + 5}$ and $g(x) = 6x - 1,$ then $(g \circ f)(-2)$ is equal to

A. 1
B. $\dfrac{16}{3}$
C. $-\dfrac{1}{8}$
D. -3

Question #26

Reference Q.11251

Given that $p(x) = 2x + 1$ and $q(x) = x^2 - 1,$ then $p(q(x))$ equals

A. $2x^2 + 1$
B. $2x^2 - 1$
C. $4x^2$
D. $4x^2 + 4x$

Question #27

Reference Q.11253

The functions f, g, and h are given by $f(x) = x^2 - 1, g(x) = 3x + 2,$ and $h(x) = |x + 2|.$ The value of $(f \circ g \circ h)(-8)$to the nearest whole number is _____.

Question #28

Reference Q.11254

The functions f and g are given by $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x + 1}.$ If $(f \circ f)(x) = (g \circ g)\left(\dfrac{1}{2}\right),$ then the value of x to the nearest tenth, is _____.