Lesson 2: Solving Trigonometric Equations Algebraically

Question #1

Reference Q.12530

Determine the solution to each of the following equations, defined on the domain $0 \leq x \leq 2\pi$, using an algebraic approach.

a. $2\sin x = -\sqrt{3}$

b. $\cot x + \sqrt{3} = 0$

c. $3\sec x + 6 = 0$

Question #2

Reference Q.12531

Write the general solutions to the equations.

a. $2\sin x = -\sqrt{3}$

b. $\cot x + \sqrt{3} = 0$

c. $3\sec x + 6 = 0$

Question #3

Reference Q.12533

Determine the general solution to the following equations where x is in degree measure. Answer to the nearest degree.

a. $\cos x = -0.639$

b. $5\csc x + 6 = 0$

Question #4

Reference Q.12534

Use an algebraic approach to solve the following equations on the specified domain.

a. $2\cos x - \sqrt{2} = 0$
for $-2\pi \leq x \leq 0$

b. $\csc x + 2 = 0$
for $2\pi \leq x \leq 6\pi$

c. $\sqrt{3}\tan x = 1$
for $-\pi \leq x \leq 3\pi$

Question #5

Reference Q.12535

Determine the general solution, in degrees, of the equation

a. $\sin x = 0$

b. $\cos x = 0$

Question #6

Reference Q.12536

The general solution to the equation $\csc A + 2 = 0$ is

A. $A = \frac{\pi}{6} + n\pi, n \in \mathbb{Z}$

B. $A = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$

C. $A = \frac{7\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi, n \in \mathbb{Z}$

D. $A = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}$

(Note: $\mathbb{Z}$ is the set of integers)

Question #7

Reference Q.12537

In simplest form, the general solution to the equation $\sqrt{3}\cot\theta - 1 = 0$

A. $\theta = \frac{\pi}{6} + n\pi, n \in \mathbb{Z}$

B. $\theta = \frac{\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}$

C. $\theta = \frac{\pi}{3} + n\pi, n \in \mathbb{Z}$

D. $\theta = \frac{\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}$

(Note: $\mathbb{Z}$ is the set of integers)

Question #8

Reference Q.12538

The only solutions to a trigonometric equation on the domain $0 \leq x \leq 2\pi$ are $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$. An equation that has these solutions is

A. $2\sin x + \sqrt{3} = 0$

B. $2\cos x + \sqrt{3} = 0$

C. $2\sin x + 1 = 0$

D. $2\cos x + 1 = 0$

Question #9

Reference Q.12539

To the nearest degree, the solution to the equation $8\cot\theta = -1$ in the interval $540^\circ \leq \theta \leq 720^\circ$, is ____.

Question #10

Reference Q.12529

Factor the following trigonometric expressions.

a. $4\sin^2\theta - \cos^2\theta$

b. $\cot^2 x - \cot x$

c. $\sin^2\theta + 3\sin\theta + 2$

d. $\sec x \sin^2 x - 0.25\sec x$

e. $\cot^2\theta - 1$

f. $\sec^4\theta - 1$

g. $4\cos^2 A - 4\cos A - 3$

h. $2\sin^2 x - 7\sin x + 6$

Question #11

Reference Q.12532

Consider the equation $2\cos^2 x + 3\cos x + 1 = 0.$

a. Use a **graphical** approach to find the solution to the equation where $0 \leq x \leq 2\pi.$Give solutions as exact values.

b. Use an **algebraic** approach to find the solution to the equation where $0 \leq x \leq 2\pi.$ Give solutions are exact values.

c. State the general solution to the equation.

Question #12

Reference Q.12570

Algebraically find the solutions to the following trigonometric equations. Give solutions as exact values.

a. $2\sin^2\theta + \sin\theta = 0$ where $0 \leq \theta \leq 2\pi$

b. $2\sin^2 x - \sin x = 1$ where $0 \leq x \leq 2\pi$

Question #13

Reference Q.12571

Use the following information to answer the next question.

A student is solving the equation $8\cos^2 x + 2\cos x - 3 = 0$ on the interval $0^\circ \leq x \leq 360^\circ$. The student's work is shown below.

$$8\cos^2 x + 2\cos x - 3 = 0$$

$$(2\cos x - 1)(4\cos x + 3) = 0$$

$\cos x = \frac{1}{2}$	or $\cos x = -\frac{3}{4}$
quadrant 1/4	quadrant 2/3
reference angle = 60°	reference angle = 139°
$x = 60^\circ$	$x = 180^\circ - 139^\circ$
$x = 360^\circ - 60^\circ$	$x = 180^\circ + 139^\circ$
$x = 60^\circ, 300^\circ$	$x = 41^\circ, 319^\circ$

$$x = 41^\circ, 60^\circ, 300^\circ, 319^\circ$$

a. Verify algebraically that $x = 60^\circ$ is a solution to the equation.

b. Show that $x = 41^\circ$ does not satisfy the equation.

c. Explain the error in the student's work and provide a correct solution to the problem.

Question #14

Reference Q.12632

Christine is determinine the roots of the equation $2\sin x\cos x = 3\sin x$ on the domain $0 \leq x \leq 2\pi.$ Her work is shown at the side.

$$2\sin x\cos x = 3\sin x$$

$$\frac{2\sin x\cos x}{\sin x} = \frac{3\sin x}{\sin x}$$

$$2\cos x = 3$$

$$\cos x = \frac{3}{2}$$

no solution

a. Is Christine correct in stating that $\cos x = \frac{3}{2}$ has no solution? Explain.

b. Use a graphical approach to show that the equation $2\sin x\cos x = 3\sin x$ on the domain $0 \leq x \leq 2\pi$ does have roots. Give solutions as exact values.

c. Identify Christine's error and provide a correct algebraic solution to the problem.

Question #15

Reference Q.12633

A trigonometric function $f(x)$ has a period of 2π radians.

a. If the roots of the equation $f(x) = 0$ on the domain $0 \le x \le 2\pi$ are $x = a, x = b,$ and $x = c,$ state the general solution to the equation $f(x) = 0.$

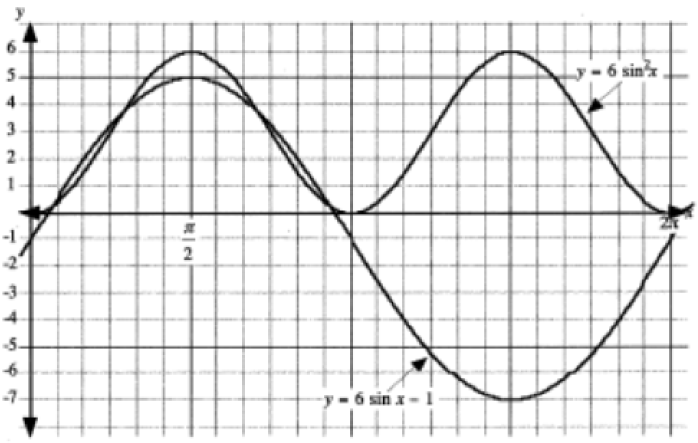
b. Use the generalization in a) and the solution in part (c) of question ID: 12632 to state the general solution to the equation $2 \sin x \cos x = 3 \cos x.$

c. The three sets of answers in b) can be simplified to a single set of answers. Write the general solution to the equation $2 \sin x \cos x = 3 \cos x$ in simplest form.

Question #16

Reference Q.12636

The diagram below shows the graphs of $y = 6 \sin^2 x$ and $y = 6 \sin x - 1$ where $0 \le x \le 2\pi.$



a. Explain how you could use this diagram to estimate the solution to the equation $6 \sin^2 x - 6 \sin x + 1 = 0,$ where $0 \le x \le 2\pi.$

b. Algebraically determine the solutions to the equation $6 \sin^2 x - 6 \sin x + 1 = 0,$ where $0 \le x \le 2\pi.$ Give the solution correct to the nearest hundredth.

c. Explain how you could use this diagram to estimate the solution to the equation $6 \sin^2 x(6 \sin x - 1) = 0,$ where $0 \le x \le 2\pi.$

d. Use an algebraic approach to find the solutions to the equation $6 \sin^2 x(6 \sin x - 1) = 0,$ where $0 \le x \le 2\pi.$ Give the solution correct to the nearest hundredth.

Question #17

Reference Q.12638

Which solutions are correct for the equation

$12 \sin^2 x - 11 \sin x + 2 = 0?$

A. $\sin x = 3, 8$

B. $\sin x = \frac{11}{12}, -2$

C. $\sin x = \frac{2}{3}, \frac{1}{4}$

D. $\sin x = -\frac{2}{3}, -\frac{1}{4}$

Question #18

Reference Q.12639

The number of solutions of the equation $2 \cos^2 x + \cos x - 1 = 0,$ where $-8\pi \le x \le 8\pi$ is ____.

Question #19

Reference Q.12640

If angle A is acute and $\log_4(\sin^2 A) = -1,$ then the value of A, to the nearest tenth of a radian, is ____.

Question #20

Reference Q.12543

a. Use an algebraic approach to solve the equation $\sin 2x = \frac{\sqrt{2}}{2}, 0 \le x \le 2\pi.$

b. State the general solution to the equation $\sin 2x = \frac{\sqrt{2}}{2}.$

Question #21

Reference Q.12544

a. Use an algebraic approach to solve the equation $\sec 3x - \sqrt{2} = 0, 0^\circ \le x \le 360^\circ.$

b. State the general solution to the equation $\sec 3x - \sqrt{2} = 0,$ where x is expressed in degrees.

Question #22

Reference Q.12545

Use an algebraic approach to determine the general solution to the equations

a. $\tan 4x = 1$

b. $\tan \frac{1}{4}x = 1$

Question #23

Reference Q.12547

In a controlled laboratory experiment, the temperature in a greenhouse is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function $C(t) = 28 + 4\sin\frac{\pi}{3}t$ where C is the temperature in degrees Celsius and t is the time in hours past midnight.

a. How many temperature cycles are there in one day?

b. Algebraically determine the temperature at 2:30 pm.

c. Algebraically determine the times during the day when then temperature is 26°.

Question #24

Reference Q.12548

The roots of the equation $2\sin^2 x - \sin x - 1 = 0,$ for $0 \le x \le 2\pi,$ are $\frac{\pi}{2}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}.$ Describe how the roots of the equation $2\sin^2\left(\frac{1}{2}x\right) - \sin\left(\frac{1}{2}x\right) - 1 = 0, 0 \le x \le 2\pi,$ relate to the roots of the equation $2\sin^2 x - \sin x - 1 = 0, 0 \le x \le 2\pi.$ Determine the roots.

Question #25

Reference Q.12549

Which of the following is NOT a solution to the equation $2\sin 3x = 0?$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{4\pi}{3}$

D. 2π

Question #26

Reference Q.12550

Which of the following equations does not have a solution in the interval $\pi \le x \le \frac{3\pi}{2}.$

A. $\csc x = -2$

B. $\cos 2x = 1$

C. $\tan\frac{1}{2}x = 1$

D. $\sin 3x = 1$

Question #27

Reference Q.12551

The smallest positive solution to the equation $\sin 4x = 0.48,$ correct to the nearest hundredth of a radian, is $x =$ ________.

Question #28

Reference Q.12525

Determine the solution to each of the following equations, defined on the domain $0 \le x \le 2\pi,$. Give solutions as exact values.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\tan x = -1$

c. $2\sec x - 4 = 0$