

Lesson 2: Transforming Exponential Equations

Question #1

Reference Q.11571

Re-write the following expression with only x in the exponent.

$$2^{2x}$$

Question #2

Reference Q.11572

Re-write the following expression with only x in the exponent.

$$2^{-x}$$

Question #3

Reference Q.11573

Re-write the following expression with only x in the exponent.

$$(8)^{\frac{x}{3}}$$

Question #4

Reference Q.11461

Are the following two exponential functions equivalent?

$$P(t) = 1000(e)^{1.14t}$$

$$P(t) = 1000(3.126768365)^t$$

Hint: e represents the constant of continuous growth:

$$2.71828182846$$

Question #5

Reference Q.11462

Are the following two exponential functions equivalent?

$$\text{Half a percent every month: } P(t) = P_0(1 + 0.005)^{12t}$$

$$6 \text{ percent every year: } P(t) = P_0(1 + 0.06)^t$$

Question #6

Reference Q.11464

How much better is the return on an 8% yearly interest rate investment that is compounded monthly as opposed to compounded yearly?

Question #7

Reference Q.11463

How much better is the return on an 8% yearly interest rate investment that is compounded daily as opposed to compounded yearly?

Question #8

Reference Q.11465

A new car usually depreciates at a rate of 15% per year.

This can be modeled by the equation $V(t) = V_0(0.85)^t$ where t is years.

You wonder to yourself, "I wonder what my new car will be worth after just a few months?"

- Estimate the monthly depreciation rate. Justify your estimate.
- What is the actual monthly depreciation rate?
- Are you surprised by the actual rate? Why or why not?

Question #9

Reference Q.12184

A computer depreciates in value very quickly, at the rate of 28% per year.

This can be modeled by the equation $V(t) = V_0(0.72)^t$ where t is years.

You wonder to yourself, "I wonder what my computer will be worth after just a few weeks?"

- Estimate the weekly depreciation rate. Justify your estimate.
- What is the actual weekly depreciation rate?
- Are you surprised by the actual rate? Why or why not?

Question #10

Reference Q.11515

A population of bacteria is doubling every 36 minutes.

- Write an equation to represent this growth.
- Use the power rule to discover the hourly rate of growth, and rewrite the equation in terms of hours.
- How many times greater will the population be after 3 hours?

Question #11

Reference Q.11516

The average half-life of caffeine in the human body is 5.7 hours.

- Write an equation to represent this decay.
- Use the power rule to discover the hourly rate of decay, and rewrite the equation in terms of hours.
- How much caffeine will still be in your system if you consumed 500mg of caffeine 10 hours ago?

Question #12

Reference Q.11570

The height of a weed is increasing continuously according to the function:

$H(t) = (3\text{cm})e^{0.13t}$ where " t " is time in days. What is the percentage growth each day?

Hint: Re-write the function in terms of "percentage growth".

$$H(t) = (3\text{cm})(1 + p)^t$$

Question #13

Reference Q.12185

Algae in a pond grows continuously according to the function:

$$A(t) = (45\text{cm}^2)e^{0.22t},$$

where " t " is time in days. What is the percentage growth each day?

Hint: Re-write the function in terms of "percentage growth".

$$A(t) = (45\text{cm}^2)(1 + p)^t$$

Question #14

Reference Q.12195

An investment of \$25,000 is invested at 3.2% per month and is compounded monthly. The growth can be modeled by the equation

$$A(t) = 25,000(1.032)^{12t}.$$

What is the equivalent annual growth rate for this investment (rounded to the nearest tenth of a percent) and what is the investment worth about 9 years?

Question #15

Reference Q.12197

An investment of \$3,000 is invested at 1.7% per month and is compounded monthly. The growth can be modeled by the equation

$$A(t) = 3,000(1.017)^{12t}.$$

What is the equivalent daily growth rate for this investment (rounded to the nearest thousandth of a percent). What is the investment worth in 2 years if it is compounded daily?

Question #16

Reference Q.11665

Simplify.

a. $49^{x-1} \times 7^{2x-3}$

b. $216^x \div (1296^{5x-4} \times 36^{x+5})$

Question #17

Reference Q.11666

Solve the following exponential equations.

a. $2^{5x+2} = 2^{17}$

b. $2^{2-3x} = 2^{7x-8}$

c. $9^{c+1} = 729$

d. $5^{4a+2} = 25^a$

e. $2^{-t} = 64$

f. $10^{10x-2} = 1000^{20x-42}$

Question #18

Reference Q.11667

Solve for x .

a. $2^x = 16\sqrt{2}$

b. $8^{3x} = 4^{1-x}$

c. $9^{3x+1} = 27^{3x}$

Question #19

Reference Q.11668

A bacterium doubles every 12 hours. The number of bacteria, N ,

present after H hours is given by the formula $N = 2^{\frac{H}{12}}$.

a. After how many hours are there 256 bacteria?

b. Estimate how many hours it would take for the number of bacteria to reach 1000?

(An algebraic technique to determine the exact answer to this problem will be shown later in this unit.)

Question #20

Reference Q.11669

A radioactive isotope has a mass of 512 grams, and has a half life of 45 minutes.

The number of grams, N , present after m minutes, is given by the

formula $N = 512\left(\frac{1}{2}\right)^{\frac{m}{45}}$.

How long would it take for the mass to reduce to one gram?

Ⓜ **Question #21**

Reference Q.11670

Solve for x .

a. $\left(\frac{4}{7}\right)^{5x} = \left(\frac{64}{343}\right)^{2x-1}$

b. $49\left(\frac{7}{12}\right)^{2x} = 144$

c. $\left(\frac{125}{216}\right)^{-\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

d. $\left(\frac{9}{4}\right)^{x+3} = \left(\frac{8}{27}\right)^{-5}$

Ⓜ **Question #22**

Reference Q.11671

Solve for x .

a. $2^{x-1} = (128^x)(2^x)$

b. $\left(\frac{1}{4}\right)^{x-12} = (2)(32)^{2x+1}$

c. $\sqrt[3]{\frac{27^{2x-1}}{3^{x+1}}} = 9$

Ⓜ **Question #23**

Reference Q.11672

Solve the equation $2(6^{2x}) - 74(6^x) + 72 = 0$.

(Hint: Write as a quadratic equation with the variable as 6^x .)

Ⓜ **Question #24**

Reference Q.11673

If $4^{2x-7} = \frac{1}{64}$, then the value of \sqrt{x} is

- A. 2
- B. $\sqrt{2}$
- C. $\sqrt{5}$
- D. $\frac{3}{2}$

Ⓜ **Question #25**

Reference Q.11674

The solution to the $25^{x+1} = 5^{3(x-1)}$, to the nearest tenth, is $x =$ _____.

Ⓜ **Question #26**

Reference Q.11675

The solution to the equation $\left(\frac{1}{8}\right)^{x-3} = (2)(16)^{2x+1}$, to the nearest hundredth, is $x =$ _____.

Ⓜ **Question #27**

Reference Q.11676

The solution to the equation $8^{2x-1} = 16$, to the nearest tenth, is $x =$ _____.