

AP Calculus AB/BC - Differentiation Properties and Methods

Show all your work. Clearly indicate the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations. Use complete sentences for all explanations.

This assignment is longer because it covers 2 units in the AP Classroom outline.

1. Use the limit definition of the derivative to find the derivative of $f(x)$ at $x = 4$.

$$f(x) = \frac{5}{\sqrt{3x+4}}$$

2. The function $f(x) = \tan(x)$ is concave up in the interval $0 < x < \frac{\pi}{2}$.

Without finding the expression of the derivative, explain how you will find an approximation for the slope of the tangent line of $f(x)$ at $x = 1$.

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3. Give an example of a (2-piece) piecewise continuous function which is also differentiable at the junction $x = 0$. Be sure to define the function and explain using complete sentences.
4. Differentiate the following functions using differentiation rules (not the limit definition). If a differentiation rule is specified, you must use that rule.

a) $f(x) = (x^2)^4$ incorporating Chain rule

b) $f(x) = \frac{\cos x}{1 + \sin x}$

c) $f(x) = \frac{\sin x}{\sqrt{1 - \cos^2 x}}$ incorporating product rule

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d) $f(x) = \left(x^{\frac{2}{3}} + 1\right)^{\frac{7}{3}}$

5. Consider the curve defined by $x^2y - xy^2 = 30$.

a) Find an expression for $\frac{dy}{dx}$

b) Write an equation for the line tangent to the curve at the point (6,1).

c) Comment on the concavity of the curve at $x = 0$.

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6. If $f(x) = \tan^{-1} 3x$, then evaluate

$$\lim_{x \rightarrow \sqrt{3}} \frac{f(x) - f(\sqrt{3})}{x - \sqrt{3}}$$

7. Let $f(x)$ be a function with $f'(x) = -2x \cos(x^2)$, and let $g(x): R \rightarrow R$ be an increasing and differentiable function with the following sample values.

x	-1	0	1	2	3
$g(x)$	2	3	6	8	12
$g'(x)$	1	1	4	2	5

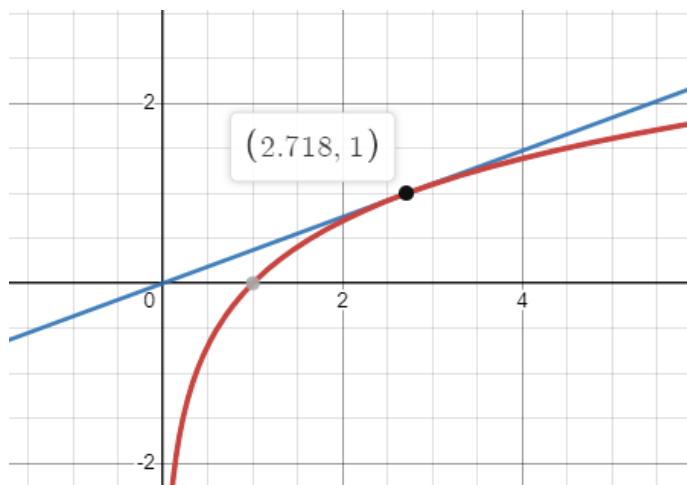
a) Write an equation for the line tangent to the graph of $g^{-1}(x)$ at $x = 6$.

b) Let $h(x) = f\left(g\left(\frac{x}{2}\right)\right) + 21$. Find $h'(4)$. Express your answer in exact form.

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8. Let $f(x) = \ln(x)$. The line through the origin tangent to $f(x)$ has slope $\frac{1}{e}$. The graph is for your reference.

- a) Find an equation for this tangent line.



- b) Explain (in complete sentences and/or math) why $\ln(x) < \frac{x}{e}$ for all positive $x < e$.
- c) Using the result above, show that $x^e < e^x$ for all positive $x < e$.
- d) Conclude which one, π^e , or e^π is larger.

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9. Let

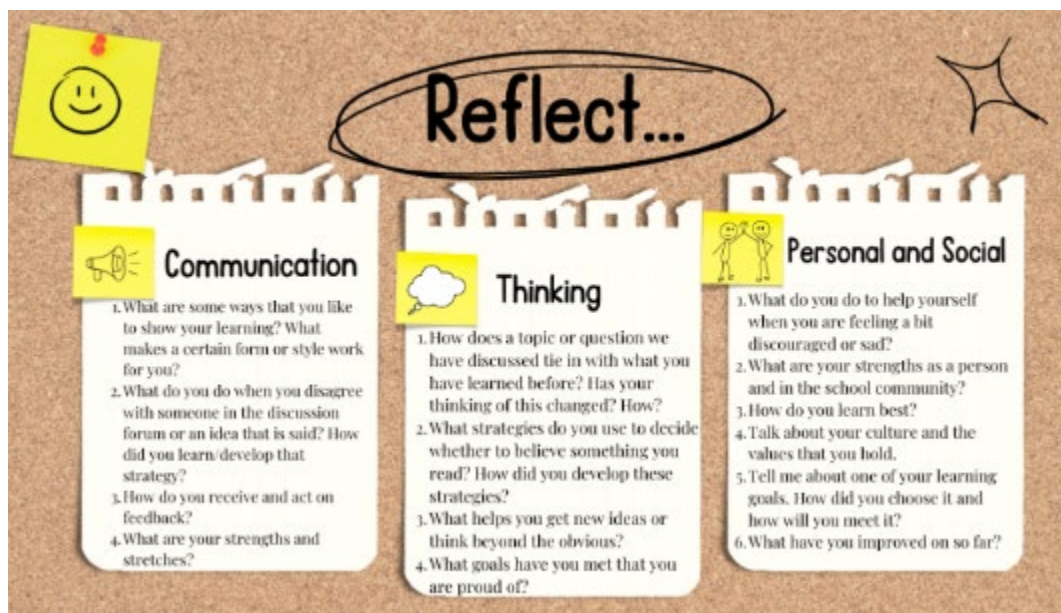
$$y = \frac{(x + 2)^5(2x - 3)^4}{(x + 12)^2}$$

a) Use the technique of logarithmic differentiation to find $\frac{dy}{dx}$ at $x = 0$.

b) Find $\frac{d^2y}{dx^2}$ at $x = 0$.

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10. Think about your learning journey in this unit. Pick a category and write a response to ONE of the category's questions in a few complete sentences.



~The End~

Quick checklist:

Did you...

- ☐ State your final answer and explanations using full sentences?
- ☐ Explain in a few sentences every time you have a Yes/No answer?
- ☐ Use equal signs correctly where necessary?