Lesson 3: Trigonometric Ratios of Any Angle

Question #1

Reference Q.12390

In which quadrant(s) does the terminal arm of rotaton angle θ lie if

a. $\sin\theta$ is negative?

b. $\sec\theta$ is positive?

c. $\csc\theta$ and $\tan\theta$ are both negative?

d. $\cot\theta$ is positive and $\csc\theta$ is negative?

Question #2

Reference Q.12391

Determine, without using technology, whether the given trigonometric ratios are positive or negative.

a. $\cos 181°$

b. $\csc\dfrac{11\pi}{6}$

c. $\tan(-300°)$

d. $\sin\dfrac{14\pi}{3}$

e. $\cot 560°$

f. $\sec\left(-\dfrac{\pi}{4}\right)$

Question #3

Reference Q.12392

Find the value (to 4 decimal places where necessary) of

a. $\tan\dfrac{\pi}{4}$

b. $\cos(-382°)$

c. $\sin\left(-\dfrac{2\pi}{3}\right)$

d. $\cot 30°$

e. $\csc 60°$

f. $\sec\left(-\dfrac{7\pi}{6}\right)$

Question #4

Reference Q.12393

Rewrite as the same trigonometric function of a positive acute angle.

a. $\sin 205°$

b. $\cot\dfrac{3\pi}{5}$

c. $\csc 107°$

d. $\sec\left(-\dfrac{19\pi}{9}\right)$

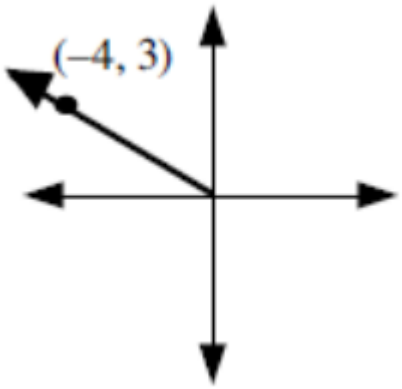
e. $\cos 5\pi$

f. $\tan(-30°)$

Question #5

Reference Q.12394

The point $(-4, 3)$ lies on the terminal arm of a rotation angle as shown.



Determine the primary and reciprocal trigonometric ratios for the rotation angle. Express each answer as an exact value.

Question #6

Reference Q.12395

The point $(10, -24)$ lies on the terminal arm of an angle θ in a standard position. Determine the exact values of $\sec\theta$ and $\csc\theta$.

Question #7

Reference Q.12396

Solve for the required in each of the following. Express each answer as an exact value with a rational denominator.

a. If $\tan\theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the first quadrant, determine $\cot\theta, \csc\theta,$ and $\sec\theta.$

b. If $\tan\theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the third quadrant, determine $\cot\theta, \csc\theta,$ and $\sec\theta.$

Question #8

Reference Q.12397

If $\sin X = -\frac{1}{3}$ and $\cos X$ is positive, express $\cot X$ as an exact value.

Question #9

Reference Q.12398

$\cos A = -0.28,$ where $\pi \le A \le \frac{3\pi}{2}.$ Determine the exact value of $\csc A.$

Question #10

Reference Q.12399

The point $\left(\frac{1}{5}, -\frac{1}{5}\right)$ lies on the terminal arm of an angle A in standard position. The exact value of $\sec A$ is

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. $\frac{\sqrt{2}}{25}$

D. $-\frac{\sqrt{2}}{25}$

Question #11

Reference Q.12400

Consider the following trigonometric expressions.

I. $\cos(2\pi + x)$

II. $\cos(2\pi - x)$

III. $\cos(\pi - x)$

IV. $\cos(-x)$

If $\cos x = A,$ which of the following is not equal to A?

A. III only

B. IV only

C. III and IV only

D. some other combination of I, II, III, and IV

Question #12

Reference Q.12401

Without using technology, determine which of the following has a different sign from the others.

A. $\tan 201^\circ$

B. $\csc(-72^\circ)$

C. $\sec 115^\circ$

D. $-\cot 79^\circ$

Question #13

Reference Q.12402

Without using technology, determine which of the following does not have the same value as $\cot 277^\circ.$

A. $\cot(-83^\circ)$

B. $\cot(-263^\circ)$

C. $-\cot 263^\circ$

D. $-\cot 97^\circ$

Question #14

Reference Q.12403

Angles A, B, and C are rotation angles with the following properties.

$\csc A = \csc\frac{\pi}{4},$ where $0 \le A \le 2\pi, A \ne \frac{\pi}{4}$

$\cot B = \cot\frac{3\pi}{4},$ where $0 \le B \le 2\pi, B \ne \frac{3\pi}{4}$

$\sec C = \sec\frac{8\pi}{5},$ where $0 \le C \le 2\pi, C \ne \frac{8\pi}{5}$

If the value of $A + B + C$ can be expressed in the form $k\pi,$ then the value of k, to the nearest hundredth, is ___.

Question #15

Reference Q.12404

The point $(4, -8)$lies on the terminal arm of angle $\theta.$ If the value of $\sin\theta + \sec\theta$ can be expressed in the form $k\sqrt{5},$ then the value of k, to one decimal place, is ___.

Question #16

Reference Q.12405

Determine the measure(s) of $\theta,$ to the nearest degree, where $0^\circ \le \theta \le 360^\circ.$

a. $\sin\theta = 0.7301$

b. $\cos\theta = -0.9580$

c. $\tan\theta = \frac{5}{2}$

d. $\sin\theta = -1$

Question #17

Reference Q.12406
Determine the measure of A, to the nearest degree, for the specified domain.

a. $\sec A = 1.2354, 0° \leq A \leq 360°$

b. $\cot A = -0.4457, 180° \leq A \leq 360°$

c. $\csc A = 1.0138, 0° \leq A \leq 180°$

d. $\cot A$ is undefined for $0° \leq A \leq 360°$

Question #18

Reference Q.12407
Solve for θ, to the nearest degree, where $0° \leq \theta \leq 360°$

a. $\tan^2 \theta = 3$

b. $\sec^2 \theta = \frac{4}{3}$

Question #19

Reference Q.12408
In each case, determine the value(s) of x to the nearest hundredth of a radian.

a. $\tan x = 0.5371, 0 \leq x \leq 2\pi$

b. $\cot x = -1.5, 0 \leq x \leq 2\pi$

c. $\csc x = 6, 0 \leq x \leq \pi$

d. $\cos x = -\frac{4}{5}, \pi \leq x \leq 2\pi$

Question #20

Reference Q.12409
In each case, determine the exact values of θ in the interval $0 \leq \theta \leq 2\pi$, for which

a. $\sin \theta = -\frac{1}{2}$

b. $\cos \theta = -\frac{1}{\sqrt{2}}$

c. $\tan \theta = -1$

d. $\cot \theta = \sqrt{3}$

e. $\csc \theta = 1$

f. $\sec \theta = 2$

Question #21

Reference Q.12410
Find the values of each angle θ if $0 \leq \theta \leq 2\pi.$

a. $\cot^2 \theta = 3$

b. $\csc^3 \theta = -8$

Question #22

Reference Q.12411
The values of x for which $\sec x = -10.366$ in the interval $0° \leq x \leq 360°$ are
A. $84°, 276°$
B. $96°, 264°$
C. $96°, 276°$
D. $264°, 276°$

Question #23

Reference Q.12412
The domain for which $\sec \theta = -3.1$ has two solutions is
A. $0 \leq \theta \leq \pi$
B. $\pi \leq \theta \leq 2\pi$
C. $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$
D. none of the above

Question #24

Reference Q.12413
Which of the following has a solution in the interval $0 \leq x \leq 2\pi$ which can be expressed as an exact multiple of π radians?
A. $\tan x = \frac{1}{2}$
B. $\cot x = \sqrt{2}$
C. $\csc^2 x = -\frac{1}{4}$
D. $\sec^2 x = \frac{4}{3}$

Question #25

Reference Q.12414
If $\csc \theta = \frac{7}{2}$, then one approximate measure in radians for θ is
A. 2.852
B. 2.897
C. 3.431
D. 5.993

Question #26

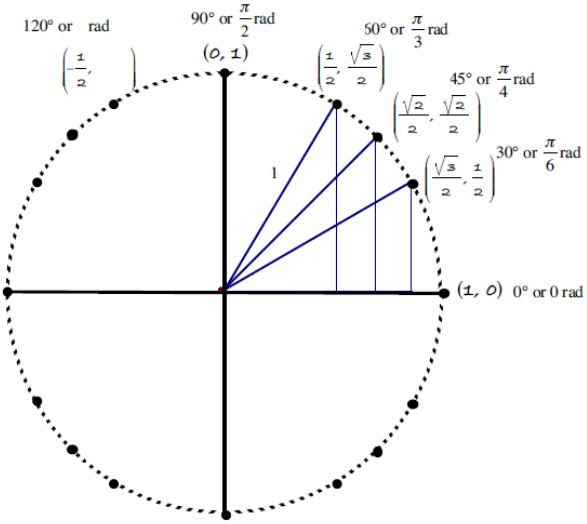
Reference Q.12415

To the nearest tenth of a radian, the value of x for which $\cot x = -\frac{1}{4}$ and $\pi \leq x \leq 2\pi$ is ___.

Question #27

Reference Q.12416

Consider the diagram below. Complete the diagram by writing the coordinates and the rotation angle (in degrees and in radians) for each point on the circumference of the circle. [Interesting Quote: "Aboriginal peoples see and respond to the world in a circular fashion and are influenced by the examples of the circles of creation in our environment." - J. Dumont]



Question #28

Reference Q.12417

Use either the unit circle or a reference triangle to determine the exact value of the following.

a. $\cos 150°$

b. $\sin 315°$

c. $\sin(-30°)$

d. $\tan 240°$

e. $\tan 480°$

f. $\cos^2 225°$

Question #29

Reference Q.12418

Determine the exact vaue of the following.

a. $\sin \frac{5\pi}{3}$

b. $\tan \frac{7\pi}{6}$

c. $\cos\left(-\frac{2\pi}{3}\right)$

d. $\sin(-\pi)$

e. $\cos\left(-\frac{5\pi}{3}\right)$

f. $\tan^2\left(\frac{2\pi}{3}\right)$

Question #30

Reference Q.12419

Determine the exact value of the following.

a. $\sec 300°$

b. $\cot \frac{5\pi}{6}$

c. $\csc\left(-\frac{5\pi}{3}\right)$

d. $\cot 930°$

e. $\sec \frac{3\pi}{2}$

f. $\csc 5\pi$

Question #31

Reference Q.12420

State the exact coorinates of the point on the unit circle that correspond to each rotation.

a. $\frac{3}{2}\pi$ radians

b. $360°$

c. $\frac{7\pi}{6}$ radians

Question #32

Reference Q.12436

Use a calculator to determine, to four decimal places, the coordinates of the point on the unit circle that corresponds to each rotation.

a. $175°$

b. $\frac{13\pi}{10}$ radians

Question #33

Reference Q.12437

The point $T(0.4695, -0.8829)$ lies on the unit circle. Determine the value of θ, in degrees, where θ is the angle made by the positive x-axis and the line passing through T.

Question #34

Reference Q.12438

$P\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point P to point Q, through what angle has it rotated? Answer in degrees and in radians.

Question #35

Reference Q.12440

Find the measure of θ where $0 \leq \theta \leq 2\pi$.

a. $\sin\theta = -\frac{\sqrt{3}}{2}$

b. $\tan\theta = 0$

c. $\tan\theta$ is undefined

Question #36

Reference Q.12441

Write the equation of the following.

a. the unit circle

b. circle with centre $(0, 0)$ and radius $\sqrt{10}$

c. circle with centre $(0, 0)$ and passing through the point $(-8, 6)$

Question #37

Reference Q.12442

The point $A\left(x, \frac{\sqrt{5}}{5}\right)$ lies on the unit circle in quadrant 2.

a. Determine the value x. Answer as a radical with a ratonal denominator.

b. The point A lies on the terminal arm of a rotation angle θ. Determine the exact values of $\tan\theta$ and $\csc\theta$.

c. Determine the value of θ to the nearest tenth of a radian of $0 \leq \theta \leq 2\pi$.

Question #38

Reference Q.12443

If the point $(-a, a\sqrt{2})$ lies on the circle with centre $(0, 0)$ and radius 12, determine all possible values of a.

Question #39

Reference Q.12444

The exact value of $\sec\left(-\frac{3\pi}{4}\right)$ can be written in the form $k\sqrt{2}$. The value of k is

A. 1
B. -1
C. 2
D. -2

Question #40

Reference Q.12445

Which of the following points does not lie on the unit circle?

A. $(-1, 0)$
B. $\left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$
C. $\left(\frac{2}{5}, \frac{3}{5}\right)$
D. $\left(-\frac{8}{17}, -\frac{15}{17}\right)$

Question #41

Reference Q.12446

The smallest value of θ for which $\cot\theta$ is undefined in the interval $[90°, 360°]$ is ___.