

AP Calculus AB/BC - Differentiation Properties and Methods

Show all your work. Clearly indicate the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations. Use complete sentences for all explanations.

This assignment is longer because it covers 2 units in the AP Classroom outline.

1. A carton of juice is taken out of the refrigerator and left on the counter on a hot summer day. Its temperature can be modeled by the function

$$f(t) = 26 - 22(0.97)^t$$

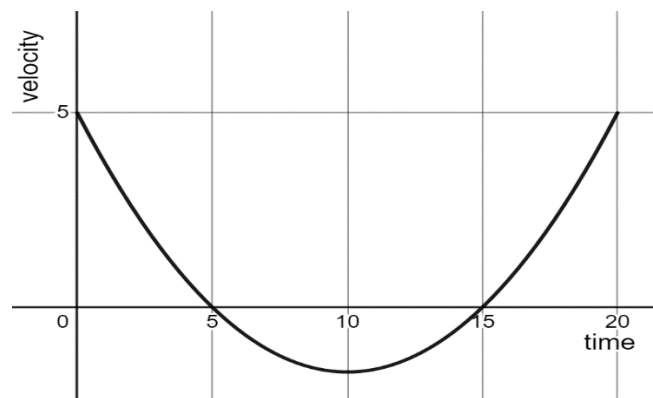
where $f(t)$ represents the temperature in Celsius and t represents the time in minutes.

- a) In complete sentences, explain what the expression $f'(30)$ means.
- b) Explain how to find the temperature of the refrigerator. You do not need to find the temperature of the refrigerator.
- c) Use correct mathematical notation to find the temperature of the room. Explain your reasoning.

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- d) When is the juice's temperature increasing the fastest? Explain your reasoning.

2. A particle traveling along the x-axis has position $x(t)$ at time t . The graph of the particle's velocity is shown for $0 \leq t \leq 20$.



Give an expression in terms of $x(t)$ (for example: $x(4) - x(2)$ or $x(5) + 1$) that shows

- a) The total displacement of the particle from $t = 0$ to $t = 20$.
- b) The total distance of the particle from $t = 0$ to $t = 20$.

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3. Let $f(x)$ be a differentiable function with $f'(x) = \sin x \cos x$. On which interval(s) within $[-\pi, \pi]$ is $f(x)$ both increasing and concave up?

4. A twice differentiable function has the following selected values.

x	-3	-2	-1	0	1	2	3	4
$f'(x)$	1	0	-0.6	0	-0.4	0	2.1	0
$f''(x)$	0	-0.5	-0.2	0	1.2	1	-0.1	-0.8

Identify any relative maxima and minima. Explain your reasoning.

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5. A kayaker leaves Deep Cove at 12:30pm, kayaks 10km, and returns at 2:30pm. Prove that there were at least two times when the kayaker was kayaking at 4km/h.
6. A milk processing plant has a vat of milk. It is filled from a line after pasteurization, then emptied into bottles. The volume of milk in the vat can be given by a twice differentiable function $V(t)$, where $t \in [0,4]$ is given in hours and $V(t)$ is given in hundreds of gallons. We know that the second derivative $V''(t)$ is negative for hours $t \in [2,4]$. Selected values are given below.

t	0	2	2.5	3	3.8	4
$V(t)$	32	29	32	33	33	31

- a) Is this data enough to conclude that, at some point in the first two hours, the rate of change of $V(t)$ was zero? Why or why not?
- b) Is this data enough to conclude that, at some point in the last two hours, the rate of change of $V(t)$ was zero? Why or why not?

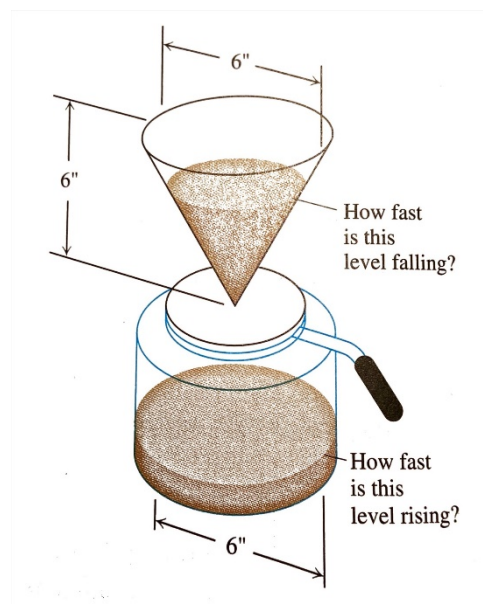
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c) Use the data to estimate the milk volume's rate of change after 2 hours and 15 minutes.

d) We have that $V'(2) = 7$. Use the locally linear approximation for V at $t = 2$ hours to approximate the amount of milk in the vat at $t = 2.25$ (2 hours and 15 minutes). Is your approximation an over-estimation or an under-estimation?

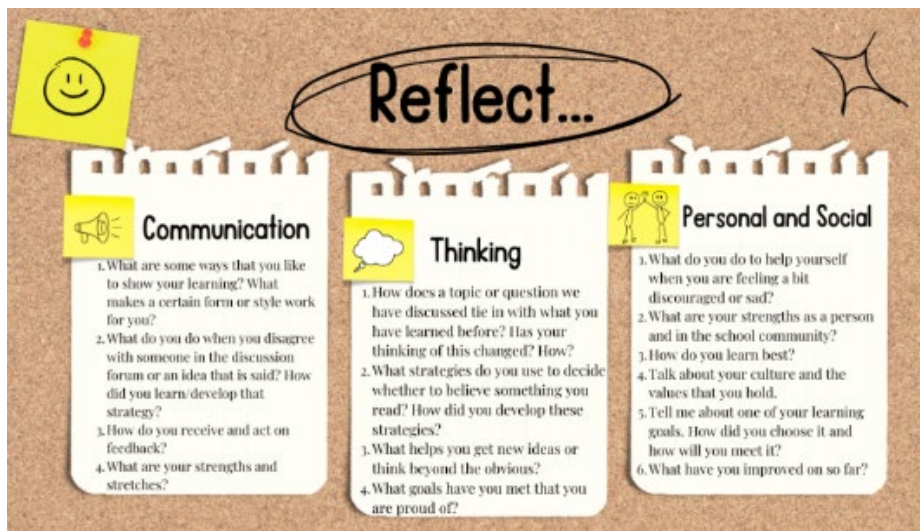
7. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of $10 \text{ in}^3/\text{min}$.

a) How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?



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- b) How fast is the level in the cone falling at that moment?
8. Think about your learning journey in this unit. Pick a category and write a response to ONE of the category's questions in a few complete sentences.



~The End~

Quick checklist:

Did you...

- ☐ State your final answer and explanations using full sentences?
- ☐ Explain in a few sentences every time you have a Yes/No answer?
- ☐ Use equal signs correctly where necessary?