Lesson 4: Sigma Notation

Question #1

Reference Q.18023

Evaluate:

a. $\sum_{k=1}^{3} k+1=$

b. $\sum_{k=1}^{5} \frac{1}{2^k}=$

c. $\sum_{k=1}^{\infty} \frac{1}{2^k}=$

Question #2

Reference Q.381

Evaluate the following to practice the basic skills of Sigma Notation.

$$\sum_{i=1}^{3} i^4$$

Question #3

Reference Q.382

Evaluate the following to practice the basic skills of Sigma Notation.

$$\sum_{k=-4}^{1} \left(2k^2-2k+1\right)$$

Question #4

Reference Q.383

Write in sigma notation (but don't evaluate):

$3+6+9+12+\ldots+21$

Question #5

Reference Q.18024

Write the first six terms of the sequence. $t_1=1, t_n=t_{n-1}+3$

Question #6

Reference Q.13029

Determine the common ratio for each of the following geometric series, and state whether a sum to infinity exits. Calculate this sum where it exists.

a. $4+2+1+\ldots$

b. $5-1+\frac{1}{5}-\ldots$

c. $-4+6-9+\ldots$

d. $1+1+1+\ldots$

e. $10-9+8.1-\ldots$

f. $1-1+1-\ldots$

g. $15-9+\frac{27}{5}-\ldots$

h. $\frac{3}{4}-\frac{3}{8}+\frac{3}{16}-\ldots$

i. $0.0001+0.001+0.01+\ldots$

j. $2^6+2^5+2^4+\ldots$

k. $2^4+2^5+2^6+\ldots$

l. $\frac{3}{100}+\frac{3}{10\,000}+\frac{3}{1\,000\,000}+\ldots$

m. $3+\sqrt{3}+1+\frac{1}{\sqrt{3}}+\ldots$

Question #7

Reference Q.13035

Consider the geometric sequence $12+6+3+\ldots$

a. Calculate, to four decimal places, the sums for 10 and 12 terms of the sequence.

b. Explain why these sums are almost equal.

c. Predict the sum for the infinite series.

d. Calculate the sum for the infinite series.

Question #8

Reference Q.13037

The first term of a geometric series is 81 and the third term is 1. Determine the sum to infinity of each of the two possible series.

Question #9

Reference Q.13039

a. Show that $4x^{\frac{4}{3}}, 8x^{-\frac{1}{3}}, \text{ and } 16x^{-2}$ could be the first three terms of a geometric series.

b. If $x = 8,$ explain why a sum to infinity exists, and determine this sum.

Question #10

Reference Q.13040

The infinite geometric series is given by

$1 - 3x + 9x^2 - 27x^3 + \ldots$.

If the infinite sum is $\frac{5}{9}$, determine the numerical value of the common ratio.

Question #11

Reference Q.13044

Use an infinite series to express the following repeating decimals as fractions.

a. $0.\overline{5}$

b. $0.\overline{35}$

c. $0.3\overline{5}$

Question #12

Reference Q.18025

Show that $0.\overline{11} = \frac{1}{9}$. (Use an infinite geometric series.)

Question #13

Reference Q.18026

Find a formula for S_n for the first n terms of

$9 - 3 + 1 - \frac{1}{3} + - \ldots$ What is the sum as n approaches infinity?

Question #14

Reference Q.13070

The common ratio of a geometric series is $-\frac{2}{3}$ and the sum to infinity is $-12.$

The second term, to the nearest tenth, is ______.

Question #15

Reference Q.13057

During the first week of operation, an oil well produced 8000 barrels of oil. The production dropped by 2% each week.

a. Calculate to the nearest barrel;
   - ii. the number of barrels produced in week 6
   - iii. the total number of barrels produced in the first ten weeks of production
   - iv. the total number of barrels which could be produced before the well runs dry

b. Why might the actual number of barrels produced differ from the answer to a) iii)?

Question #16

Reference Q.13066

The sum of the infinite geometric series $t + t^2 + t^3 + t^4 + \ldots$ is $4t,\ t \neq 0.$ The value of t is

A. $\frac{4}{3}$
B. $\frac{3}{4}$
C. $\frac{1}{2}$
D. $\frac{1}{4}$

Question #17

Reference Q.13068

An expression for the sum of the infinite series

$x^2 + 1 + \frac{1}{x^2} + \frac{1}{x^4} + \cdots$ is

A. $\frac{x^4}{x^2 - 1}$
B. $\frac{x^4}{x^2 + 1}$
C. $\frac{1}{x^2 - 1}$
D. x^4

Question #18

Reference Q.13072

The third term of a geometric series is $\frac{4}{3}$, and the sixth term is $\frac{32}{81}$.

The difference between the sum of the first five terms and the sum to infinity of the series , correct to the nearest tenth, is ______.

Question #19

Reference Q.18027

You drop a ball from a height of 67 inches and the ball starts bouncing. After each bounce, the ball reaches a height of 80% of the previous height. Write a rule for the height of the ball after the n^{th} bounce. Then, find the height of the ball after the 6^{th} bounce.