# Lesson 4: Solving Equations Using Logarithms

## Question #1

*Reference Q.11388*
Solve using the power rule.
$6^{8x} = 2$

## Question #2

*Reference Q.11389*
Solve using the power rule.
$0.4^{3x} = 7$

## Question #3

*Reference Q.11390*
Solve using the power rule
$8^{-0.3x} = 11$

## Question #4

*Reference Q.11391*
Solve using the power rule.
$-0.1^{4x} = 20$

## Question #5

*Reference Q.11392*
Solve using the power rule.
$\frac{7}{8}^{-12x} = 27$

## Question #6

*Reference Q.11393*
Determine the value of $x$ to 2 decimal places.
$5^{3x} = 2^{x-1}$

## Question #7

*Reference Q.11397*
Determine the value of x to 2 decimal places
$1.8^{x-3} = 3^{4x}$

## Question #8

*Reference Q.12200*
What is the exact value of x in the exponential equation
$-2 + e^{4x} = 19$?

## Question #9

*Reference Q.12203*
What is the exact value of x in the exponential equation
$11 + e^{0.2x} = 18$?

## Question #10

*Reference Q.12205*
What is the value of x in the exponential equation
$-3 + e^{-2.2x} = -2$?

## Question #11

*Reference Q.11399*
Try it!
If $\log_4 68 = x + y$ and $\log_2 32 = x - y$, what are x and y?

## Question #12

*Reference Q.11400*
The population of mice grows according to the equation
$m(t) = m_o(3)^{\frac{t}{2}}$
where t is the time in hours. The initial population is 23 mice. How many hours from now does the population increase to 89 mice?

## Question #13

*Reference Q.11401*
In the last 34 years the population of Black Rhinos at one point was decreasing at a rate of 76% according to the formula:
$P(t) = 65,000(0.24)^t$
with t is time in years.

a. What is the initial population of rhinos?

b. In how many years will there be 10 rhinos remaining?

## Question #14

*Reference Q.11696*

Express each of the following in logarithmic form.

a. $5^2 = 25$

b. $3^0 = 1$

c. $2^{-4} = \frac{1}{16}$

d. $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

e. $b^d = e$

## Question #15

*Reference Q.11700*

By converting to exponential form, solve the equation $\log_2(\log_2(x-7)) = 3.$

## Question #16

*Reference Q.11697*

Express each of the following in exponential form.

a. $\log_3 9 = 2$

b. $\log_5 625 = 4$

c. $\log_4 \frac{1}{4} = -1$

d. $\log_a f = i$

e. $\log_{10} 0.001 = -3$

## Question #17

*Reference Q.11698*

Is $y = \log_3 x$ the logarithmic form of $y = 3^x$? Explain your answer.

## Question #18

*Reference Q.11699*

Complete the following table:

| Logarithmic Form | Exponential Form | Value of $x$ |
|---|---|---|
| $\log_4 x = 2$ | | |
| | $7 = 49^x$ | |
| $\log_x\left(\frac{1}{64}\right) = -3$ | | |
| | $x + 2 = 4^2$ | |
| $\log_{32} x = \frac{1}{5}$ | | |
| | $\frac{1}{2} = 16^x$ | |

## Question #19

*Reference Q.11701*

Determine the inverse of the following functions. Answer in the form $y =$ ____.

a. $y = 3^x$

b. $y = \log_4 x$

c. $y = 3x^2 + 2$

d. $y = \log_3 x$

e. $y = 20^x$

f. $x = 20^y$

## Question #20

*Reference Q.11702*

Change each of the following from exponential form to logarithmic form.

a. $y = 3(2)^x$

b. $y = 10(3)^x$

c. $y = \frac{5}{6}(10)^x$

d. $a = b(c)^d$

## Question #21

*Reference Q.11703*
Change each of the following from logarithmic form to exponential form $y = ab^x$.

a. $\log_8\left(\frac{y}{9}\right) = x$

b. $\log_{20}(6y) = x$

c. $\log_e\left(\frac{y}{5}\right) = x$

d. $\log_{10}(0.5y) = x$

## Question #22

*Reference Q.11704*
By converting to exponential form, solve the following equations for y.

a. $3 = \log_2\left(\frac{y}{4}\right)$

b. $\log_2\left(\frac{y}{5}\right) = -3$

c. $2 = \log_4 32y$

## Question #23

*Reference Q.11705*
If $\log_4(4096x) = 64$, then the value of x is

A. $4^{\frac{32}{3}}$
B. $4^{58}$
C. $4^6$
D. $4^{32}$

## Question #24

*Reference Q.11709*
Evaluate

a. $\log_{10} 1000$

b. $\log_{12} 144$

c. $\log_6 36$

d. $\log_{36} 6$

e. $\log_5 \sqrt{5}$

f. $4\log_{10} 0.001$

g. $\log_2 \sqrt{\frac{1}{512}}$

h. $-4\log_8 8^{-4}$

## Question #25

*Reference Q.11711*
Solve for x.

a. $\log_x 125 = 3$

b. $\log_{125} 5 = x$

c. $\log_4 x = -8$

## Question #26

*Reference Q.11712*
Evaluate each of the following logarithms.

a. $\log 100$

b. $\log 10^6$

c. $\log \sqrt{10}$

d. $\log 0.01$

## Question #27

*Reference Q.11713*
Evaluate the following logarithms to the nearest tenth.

a. $\ln 20$

b. $\log_e 8$

c. $\ln e^2$

## Question #28

*Reference Q.11714*

Convert the following logarithms to the base indicated.

a. $\log_8 35$ to base 7

b. $\log \frac{1}{2}$ to base 6

c. $\log_3 50$ to base e

## Question #29

*Reference Q.11715*

Evaluate, to the nearest nearest hundredth, using the change of base identity.

a. $\log_5 17$

b. $\log_{0.5} 5.9$

c. $\frac{1}{\log_5 3}$

d. $-2\log_{12} 6$

e. $\log_8 8$

## Question #30

*Reference Q.11719*

Which of the following has a negative value?

A. $-\log_4(0.1)$

B. $\log_4\left(\frac{5}{2}\right)$

C. $\log_{\frac{1}{2}}\left(\frac{2}{3}\right)$

D. $\log_4\left(\frac{2}{3}\right)$

## Question #31

*Reference Q.11721*

The value of the expression $\log_{\sqrt{2}} 8 + 2\log_9 3,$ to the nearest tenth, is ___.

## Question #32

*Reference Q.11722*

Given the equation $\log_7 x = \log_4 60,$ the value of x to the nearest whole number is ___.

## Question #33

*Reference Q.11723*

If $\log_x 27 = \log_{12} 3,$ the value of x to the nearest whole number is ___.

## Question #34

*Reference Q.11804*

Determine the exact value of x in the form $\frac{\log M}{\log N}$.

a. $5^{x-3} = 40$

b. $0.5^{x+2} = 6^{x-1}$

c. $2^{2x} = 6^{x-3}$

## Question #35

*Reference Q.11805*

Algebraically determine, to the nearest hundredth, the solution to the equation $3(2^x) = 6^{x-2}$.

## Question #36

*Reference Q.11812*

The number, N, of throat swab bacteria being grown in a culture after t hours, is given by the formula $N = N_0(10^{0.43t})$, where $N_0$ is the original number of bacteria. If there are initially 500 bacteria in the culture, determine how long it would take, to the nearest tenth of an hour, for the number of bacteria to grow to 1 million.

## Question #37

*Reference Q.11810*

A truck bought for $35,000 depreciates at a rate of 12% per year.

a. If the value of the truck is an exponential function of time, state the base of the exponential function.

b. Write an equation to represent the value, V, of the truck after t years.

c. Determine, to the nearest hundred dollars, the value of the truck after t=4 years.

d. How many years, to the nearest tenth, would it take for the value of the truck to reduce to one quarter of its purchase price?

## Question #38

*Reference Q.11813*

In 2008 the world population was approximately 6.7 billion and was increasing at an annual rate of 1.3%.

a. If the function representing the population, in billions, is of the form $y = ab^x$, state values for a and b.

b. Write an equation to represent the world population, W billions, as a function of the numbers of years, n, since 2008.

c. Assuming the same growth rate, determine, to the nearest tenth of a billion, the expected world population in the year 2025.

d. If the population continues to grow at this rate, determine the number of years, to the nearest year, for the population to double from its 2008 size.

e. Estimate the world population in 1950. State any assumptions you have made. How does your answer compare with the actual world population in 1950? Give a reason for any discrepancy.

## Question #39

*Reference Q.11815*

The value of a type of robotic technology depreciates 25% per year.

a. Write an exponential function to represent the value of this robotic technology after t years.

b. How many years, to the nearest year, would it take for the value of the robotic technology, which initially cost $575,000, to depreciate to $25,000?

## Question #40

*Reference Q.11817*

A quantity of water contains 500g of pollutants. Each time the water passes through a filter, 18% of the pollutants are removed. How many filters are needed to reduce the mass of pollutants to less than 150g?

## Question #41

*Reference Q.11823*

The population of germs in a dirty bathtub doubles every 20 minutes. How long, to the nearest minute, would it take for the population triple?

## Question #42

*Reference Q.11826*

The tripling period, to the nearest tenth of an hour, of a bacteria culture which grows from 500 cells to 64,000 cells in 50 hours is ____.

## Question #43

*Reference Q.11827*

Radioactive material decays to 40% of its original mass in 5 years. The half-life of the radioactive material, to the nearest hundredth of a year, is ____.