

Lesson 4: Trigonometric Identities - An Introduction

Question #1

Reference Q.12552

Verify the following identities for the given value of the variable.

a. $\cot x = \frac{\cos x}{\sin x}$ for $x = 60^\circ$

b. $\sin^2 x + \cos^2 x = 1$ for $x = \frac{\pi}{4}$

Question #2

Reference Q.12553

Verify the identity $1 + \cot^2 x = \csc^2 x$ for the given values:

a. $x = \frac{\pi}{6}$

b. 120°

Question #3

Reference Q.12554

Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.

Question #4

Reference Q.12556

Two major categories of trigonometric identities are listed below:

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Use the quotient identities or the Pythagorean identities to state whether the following are true or false.

a. $\cos^2 x = 1 + \sin^2 x$

b. $(\sin x)(\csc x) = 1$

c. $\sin x = \pm \sqrt{1 - \cos^2 x}$

d. $(\tan x)(\cot x) = 1$

e. $\tan^2 x - \sec^2 x = 1$

Question #5

Reference Q.12558

Write each expression as a single trigonometric ratio or as the number 1.

a. $\sin^2 x - 1$

b. $\frac{\cos t}{\sin t}$

c. $\frac{1}{\sec \theta}$

d. $(\sec t)(\sin^2 t)(\csc t)$

e. $\csc^2 x - \cot^2 x$

f. $\sin \theta + (\cot \theta)(\cos \theta)$

Question #6

Reference Q.12572

Factor to write each in a simpler form.

a. $\sec x \sin^2 x - \sec x$

b. $\sin^4 \theta - \cos^4 \theta$

Question #7

Reference Q.12573

Very often in proving identities it is simpler to try to express each side in terms of only **sin x**, **cos x**, or both. Express each of the following in terms of only **sin x**, **cos x**, or both.

- $\tan^2 x$
- $\frac{\tan x}{\sin x}$
- $\frac{\tan x}{\csc x}$
- $\frac{1}{1 + \cot^2 x}$
- $\csc x - \sin x$
- $1 - \csc^2 x$
- $\frac{1 + \cot^2 x}{\sec^2 x}$
- $\frac{\cos^2 x - 1}{\tan x}$
- $\frac{1 + \cot^2 x}{\cot^2 x}$

Question #8

Reference Q.12574

$\frac{\cos x}{1 - \sin^2 x}$ is equal to

- sec x**
- csc x**
- sin x**
- tan x**

Question #9

Reference Q.12575

$\frac{\tan^2 x + 1}{\sec x}$ is equal to

- sec x**
- csc x**
- sin x**
- tan x**

Question #10

Reference Q.12576

$\frac{\csc x}{\cot x}$ is equal to

- cos x**
- sin x**
- sec x**
- tan x**

Question #11

Reference Q.12577

Which is not an identity?

- $\cos^2 x + \sin^2 x = 1$
- $\sin x + \cos x = 1$
- $\sec^2 x - \tan^2 x = 1$
- $\tan x \cot x = 1$

Question #12

Reference Q.12578

$\sec x - \cos x$ is equal to

- $\frac{1 - \cos x}{\cos x}$
- $\frac{1 - 2 \cos x}{\cos x}$
- $\sin^2 x$
- $\sin x \tan x$

Question #13

Reference Q.12579

The expression $\frac{\tan A \cos^2 A}{\sec A}$, expressed in terms of **sin A**, is

- $\frac{\sin A}{1 - \sin^2 A}$
- $\frac{1 - \sin^2 A}{\sin A}$
- $\sin^2 x$
- $\sin A - \sin^3 A$

Question #14

Reference Q.12580

If $\tan x \neq 0$, $\cos x \neq 0$, $\cot x \neq 0$, then $\frac{1}{\tan x \cos x \cot x}$ is equal to

- $\frac{1}{\sin x}$
- sin x**
- $\frac{1}{\cos x}$
- cos x**

Question #15

Reference Q.12581

If $\sin x \neq 0$, $\cos x \neq 0$, then $\frac{\tan x \cos x}{3 \sec x \cot x}$ is equal to

- A. $\frac{1}{3}$
- B. 3
- C. $\frac{1}{3} \sin^2 x$
- D. $\frac{1}{3} \csc^2 x$

Question #16

Reference Q.12582

When verifying the identity $\cot^2 x + 1 = \csc^2 x$ for $x = \frac{\pi}{7}$, the value of the identity, to the nearest tenth, is ____.

Question #17

Reference Q.12560

Consider the statement

$$\frac{\cos x - \sin x}{\cos x} = \sin^2 x - \tan x + \cos^2 x.$$

- a. Verify the statement is true for $x = \frac{\pi}{4}$.
- b. Prove the statement is an identity.
- c. State, and give reasons for, any restrictions.

Question #18

Reference Q.12562

In each of the following

- i) verify the possibility of an identity using a graphing calculator
- ii) prove the identity using an algebraic approach and state any restrictions.

a. $\frac{\tan \theta \cos \theta}{\sin \theta} = 1$

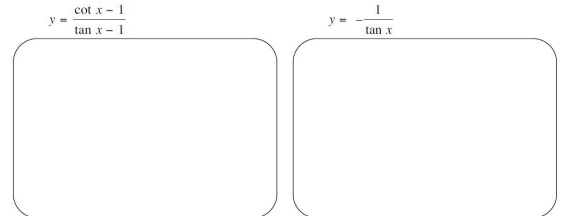
b. $\sec^2 x - \sin^2 x = \cos^2 x + \tan^2 x$

Question #19

Reference Q.12563

Consider the statement $\frac{\cot x - 1}{\tan x - 1} = -\frac{1}{\tan x}$.

- a. Verify the statement is true for $x = \frac{\pi}{3}$
- b. Prove the statement is an identity.
- c. Determine the non-permissible values of the identity.
- d. Show the non-permissible values on a graph of each side of the identity for the domain $0 \leq x \leq 2\pi$.



Question #20

Reference Q.12561

Prove the following identities using an algebraic approach.

- a. $(1 - \cos^2 x)(\csc x) = \sin x$
- b. $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$
- c. $\frac{1 - \cos x}{\sin x} = \frac{\tan x - \sin x}{\tan x \sin x}$
- d. $\frac{2}{1 - \sin x} + \frac{2}{1 + \sin x} = 4 \sec^2 x$
- e. $\frac{1 + \cos x}{\tan x + \sin x} = \cot x$
- f. $\sec x - \cos x = \frac{\sin x}{\cot x}$

Question #21

Reference Q.12564

The identity $\frac{\sec x + 1}{\sec x - 1} + \frac{\cos x + 1}{\cos x - 1} = 0$ has restrictions.

- A. $x \neq 2n\pi, \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$
- B. $x \neq 2n\pi, \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
- C. $x \neq \pi + 2n\pi, \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$
- D. $x \neq 2n\pi, n \in \mathbb{Z}$

⊙ **Question #22**

Reference Q.12565

The value of n , to the nearest tenth, for which the statement below is an identity, is _____.

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{n}{\cos x}.$$

⊙ **Question #23**

Reference Q.12566

If $\frac{p}{2} \cos^2 \frac{\pi}{5} + \frac{p}{2} \sin^2 \frac{\pi}{5} = 4$, the value of p , to the nearest tenth, is _____.