Lesson 5: Product and Quotient Laws

Question #1

Reference Q.11724

Without using a calculator, evaluate each of the following.

a. $\log_4 8 + \log_4 0.5$

b. $\log_5 100 - \log_5 4$

c. $\log_6 9 + \log_6 8 - \log_6 2$

d. $\log 2 + \log 10 - \log \frac{1}{5}$

e. $\log 8 - \log \frac{2}{5} + \log 5$

f. $\log 3 + \log 4 + \log \frac{1}{2} + \log \frac{1}{6}$

Question #2

Reference Q.11725

In each case, use laws of logarithms to write each expression as a single logarithm and evaluate for the given value of the variable.

a. $\log_x \left(\frac{4}{3}\right) + \log_x 768,$ for $x = 2$

b. $\log_a \left(\frac{7}{2}\right) - \log_a 56,$ for $a = 4$

c. $\log_b 9 - \log_b \left(\frac{1}{3}\right),$ for $b = 3$

d. $\log_n 3 + \log_n 2 - \log_n 27 - \log_n 6,$ for $n = 3$

Question #3

Reference Q.11726

Use the laws of logarithms to identify which of the following statements are true for logarithms to every base. Do not use a calculator.

a. $\log_b 2 + \log_b 3 = \log_b 5$

b. $\log_b 3 + \log_b 4 = \log_b 12$

c. $\log_b 8 = \log_b 4 + \log_b 2$

d. $\log_b 10 + \log_b 10 = \log_b 100$

e. $\log_b 2 \times \log_b 3 = \log_b 6$

f. $\frac{\log_b 8}{\log_b 2} = \log_b 4$

g. $\log_b 3^2 + \log_b 3^{-2} = 0$

h. $\log_b \frac{5}{3} = \frac{\log_b 5}{\log_b 3}$

i. $\log_b \frac{1}{8} = -\log_b 8$

Question #4

Reference Q.11727

a. Determine the value of $\log_2 p - \log_2 q$ if $\frac{p}{q} = 64.$

b. Determine the value of $4 \log_3 a + 4 \log_3 b$ if $ab = 81.$

c. Determine the value of $5 \log_5 Q - 5 \log_5 R$ if $Q = 5R.$

Question #5

Reference Q.11728

Without using a calculator, evaluate each of the following.

a. $\log_2 8^{15}$

b. $\log_7 49^{20}$

c. $\log_{49} 7^{20}$

d. $\log 10^{15}$

Question #6

Reference Q.11729

Use the laws of logarithms to identify which of the following statements are true for logarithms to every base. Do not use a calculator.

a. $\log 5^{-2} = -2 \log 5$

b. $\log 4 = \frac{2}{3} \log 8$

c. $\log 125 = \frac{3}{2} \log 25$

d. $\frac{1}{3} \log 11 = \log \frac{11}{3}$

e. $\log 5 = \frac{1}{2} \log 10$

f. $\log 2 - \log \sqrt{2} = \log \sqrt{2}$

g. $\log \frac{1}{5} - \log 5 = -\log 25$

h. $\frac{\log \sqrt{2}}{\log \sqrt{8}} = \frac{1}{3}$

Question #7

Reference Q.11730

a. Explain why $\log 81 = 4 \log 3.$

b. Hence simplify:

iii. $\log 81 - \log 27$

iv. $\frac{\log 81}{\log 27}$

Question #8

Reference Q.11731

Determine the greatest of $\frac{1}{3} \log x, \frac{2}{3} \log x, \frac{4}{3} \log x$ if

a. $x = 2$

b. $x = 1$

c. $x = \frac{1}{2}$

Question #9

Reference Q.11732

State the value of the following without the use of a calculator.

a. $\log_5 5^7$

b. $2^{\log_2 6}$

c. $\ln e^4$

d. $\log_c c^t$

e. $e^{\ln 7}$

Question #10

Reference Q.11733

Three students were asked to find an alternative expression for $\log\left(\frac{1}{x}\right), x > 0.$ Alex gave the answer as $-\log x,$ Bahman gave the answer as $\log(-x),$ and Connor gave the answer as $\log(x^{-1}).$

The correct alternative was given by

A. Connor only

B. Alex and Connor only

C. Bahman and Connor only

D. some other combination of the students

Question #11

Reference Q.11734

$\log x + \log(x + 4)$ is equal to

A. $\log(2x + 4)$

B. $\log(x^2 + 4x)$

C. $\log(x^2 + 4)$

D. $\log(x) \log(x + 4)$

Question #12

Reference Q.11735

$\log(x^2 - 4) - \log(x - 2)$ is equal to

A. $\log(x + 2)$

B. $\log(x^2 - x - 2)$

C. $\log(x - 2)$

D. $\frac{\log(x^2 - 4)}{\log(x - 2)}$

Question #13

Reference Q.11736

$(\log 2x)^2$ is equivalent to

A. $2\log 2x$

B. $\log 4x^2$

C. $2\log 4x$

D. $(\log 2)^2 + 2\log 2\log x + (\log x)^2$

Question #14

Reference Q.11737

The value of $\dfrac{3^{\log_2 4\sqrt{5}}}{3^{\log_2 \sqrt{5}}}$ to the nearest tenth is ___.

Question #15

Reference Q.11738

Write each expression as a single logarithm:

a. $\log x - 3\log y - 2\log z$

b. $\dfrac{1}{3}\log_a p + 3\log_a q - 4\log_a p$

Question #16

Reference Q.11739

Simplify the following without using a calculator.

a. $\log 2 + 2\log 3 - \log 18$

b. $2\log_4 2 - 2\log_4 4 - \log_4 \dfrac{1}{4}$

Question #17

Reference Q.11740

Use the laws of logarithms to simplify and evaluate the following expressions.

a. $\log_2 \sqrt{6} - \dfrac{1}{2}\log_2 3$

b. $\dfrac{1}{2}\log_{10} 10 + 3\log_{10} \sqrt{10}$

Question #18

Reference Q.11741

Simplify the following:

a. $\log x^4 - 3\log x + \log \dfrac{1}{x}$

b. $\log x^{\frac{1}{2}} + \log y^{\frac{1}{2}} - \dfrac{1}{2}\log xy$

c. $\log_a a^{2x+1} - \log_a a^{x-7}$

d. $\log_2 a^{x+5} + 2\log_2 a^{x-3}$

Question #19

Reference Q.11742

Show that $\log_a y^{2x-3} + \log_a y^{5x-2} - \log_a y^{x-5} - 2\log_a y^{3x+1}$ can be written as $\log_a \left(\dfrac{1}{y^2}\right)$

Question #20

Reference Q.11743

Determine the value of the following.

a. $(5^{\log_5 2})(5^{\log_5 3})$

b. $\dfrac{\left(\sqrt{2}^{\log_6 27}\right)\left(\sqrt{2}^{\log_6 16}\right)}{\sqrt{2}^{\log_6 12}}$

Question #21

Reference Q.11744

The expression $3\log_x 4 + \log_x 8 - \dfrac{1}{4}\log_x 16,$ where $x > 0,$ is equal to

A. $\log_x 384$

B. $\dfrac{3}{4}\log_x 512$

C. $\log_x 256$

D. $\dfrac{1}{4}\log_x \left(\dfrac{1}{2}\right)$

Question #22

Reference Q.11745

$\log_p(p^6q^2) - \log_p(p^2q^2)$ is equivalent to

A. 3
B. 4
C. $4p$
D. p^4

Question #23

Reference Q.11746

If $\log_3 A = t$, then $\log_3 27A^3 =$

A. $3 + 3t$
B. $3 + t^3$
C. $9t^2$
D. $3t^3$

Question #24

Reference Q.11747

If $\log_3 x^2 = 2$ and $2\log_k \sqrt{x} = \frac{1}{3}$, then the value of k is _____.

Question #25

Reference Q.11748

If $\log_3 x^2 = 4, \log_2 y^3 = 6,$ and $\log_b x + \log_b y = \frac{1}{2},$ where $x, y > 0,$ then the value of b is _____.

Question #26

Reference Q.11828

Solve for the variable in each equation. Remember to check for extraneous solutions.

a. $\log_3 x + \log_3 3 = \log_3 30$

b. $\log_3 3y - \log_3 4 = \log_3 6$

c. $2\log y = \log 25$

Question #27

Reference Q.11829

Solve for x.

a. $\log_9 x - \log_9 3 = 1$

b. $\log_4(x-5) + \log_4(x-2) = 1$

Question #28

Reference Q.11830

Solve for the variable in each equation.

a. $\log_5 x - \log_5(x-1) = \log_5 3$

b. $\log_3(3x-1) - \log_3(x-1) = 4$

c. $\log(2x+3) + \log(x+2) - 1 = 0$

Question #29

Reference Q.11831

State each logarithmic equation for the given variable. State, and explain the reason for, any extraneous roots.

a. $\log_{49}(m+4) + \log_{49}(m-2) = \frac{1}{2}$

b. $\log_8(-x) + \log_8(3-x) = \log_8 10$

c. $\log_5(7x-1) - \log_5 x = \log_5 4$

d. $\log_2 3a + \log_2 2 = \log_2 8 - \log_2 4$

e. $\log_2 x = 2 + \frac{1}{2}\log_2(x-3)$

Question #30

Reference Q.11840

The number of students in a school t years after the school opens can be modelled by the equation $S = S_0[\log_2(t+1)+1],$ where S_0 is the original number of students in the school.

a. If there were initially 100 students in the school, how many would be expected after 10 years?

b. How many years will it take for the number of students to reach 800 if the original number of students in the school was 200?

Question #31

Reference Q.11841

Determine the root(s) of the following equations.

a. $\frac{1}{2}\log_4(y+4) + \frac{1}{2}\log_4(y-4) = \log_4 3$

b. $\log_2(1-w) - \log_2(3-w) = -1$

Question #32

Reference Q.11842
Solve for x. State, and explain the reason for, any extraneous roots.

a. $\log_5(\log_x(2x-3))=0$

b. $\log_2(\log_x(20-x))=1$

c. $\log_3(\log_2(x^2-2x))=1$

Question #33

Reference Q.11843
Solve.

a. $\log x+(\log x)^2=0$

b. $(\log x)^2-\log x^5=14$

Question #34

Reference Q.11844
Solve each logarithmic equation.

a. $(\log x)^2+\log x^{-1}-12=0$

b. $2(\log_3 n)^3-(\log_3 n)^2=0$

c. $3(\log_3 x)^2-36=\log_3 x^{23}$

Question #35

Reference Q.11845
If $\log_4(2x+1)+\log_4(x-1)=\frac{1}{2},$ then the value(s) of x is/are

A. $\frac{3}{2},-1$
B. $\frac{3}{2}$ only
C. 3 only
D. $\frac{1}{2},3$

Question #36

Reference Q.11846
The equation $2\log x-\log 25=\log 9+\log x, x>0,$ has an integral solution. The value of x is ____.

Question #37

Reference Q.11847
If $\frac{1}{3}\log\sqrt{x}+\log x^3=n\log x$ for all values of x, then the value of n, to the nearest tenth, is ____.