

Lesson 5: Sketching Polynomials by Finding Zeros

Question #1

Reference Q.11340

For which of the following functions will the domain be $x \in \mathbb{R}$ (all real numbers)?

- a. $f_1(x) = x^4 - x^3 + 7x^2 + x - 8$
- b. $f_2(x) = x^3 + 2x^2$
- c. $f_3(x) = -7x^5 - 10x^4 + 26x^3 - 10x^2 + 39x - 19$
- d. $f_4(x) = 3x^3 + 17x^4 - x^2$

Question #2

Reference Q.11341

For which of the following functions will the range be $y \in \mathbb{R}$ (all real numbers)?

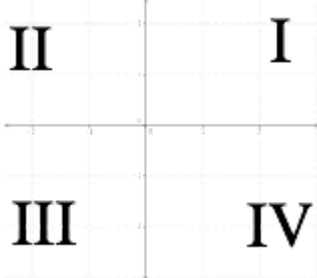
- a. $f_1(x) = x^4 - x^3 + 7x^2 + x - 8$
- b. $f_2(x) = x^3 + 2x^2$
- c. $f_3(x) = -7x^5 - 10x^4 + 26x^3 - 10x^2 + 39x - 19$
- d. $f_4(x) = 3x^3 + 17x^4 - x^2$

Question #3

Reference Q.11342

Which quadrants does the following polynomial extend to and from when graphed? $f(x) = 2x^3 - 4x^4 + 9x - 7$

Hint: The picture below shows the names of the quadrants:

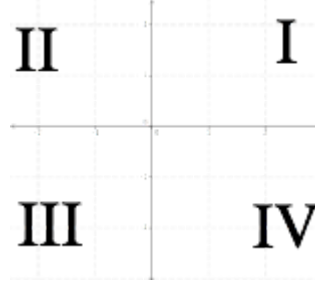


Question #4

Reference Q.11345

Which quadrants does the following polynomial extend to and from when graphed? $f(x) = x^3 - 7x^2 + x + 19$

Hint: The picture below shows the names of the quadrants:

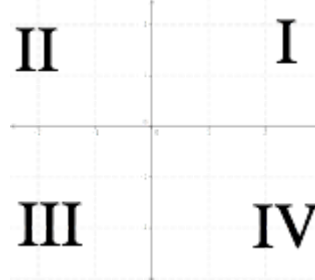


Question #5

Reference Q.11344

Which quadrants does the following polynomial extend to and from when graphed? $f(x) = (x - 6)(x - 9)(-x + 3)$

Hint: The picture below shows the names of the quadrants:

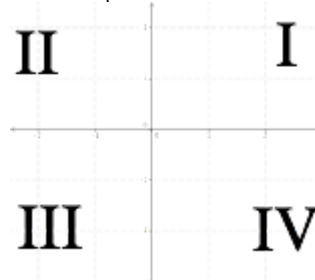


Question #6

Reference Q.11346

Which quadrants does the following polynomial extend to and from when graphed? $f(x) = -5x^2 + 7x + 1$

Hint: The picture below shows the names of the quadrants:



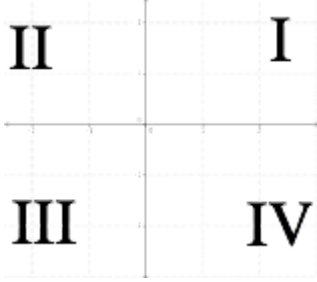
Question #7

Reference Q.11347

Which quadrants does the following polynomial extend to and from

when graphed? $f(x) = \frac{1}{972}x^{256} + 9x^{172} + x^{79}$

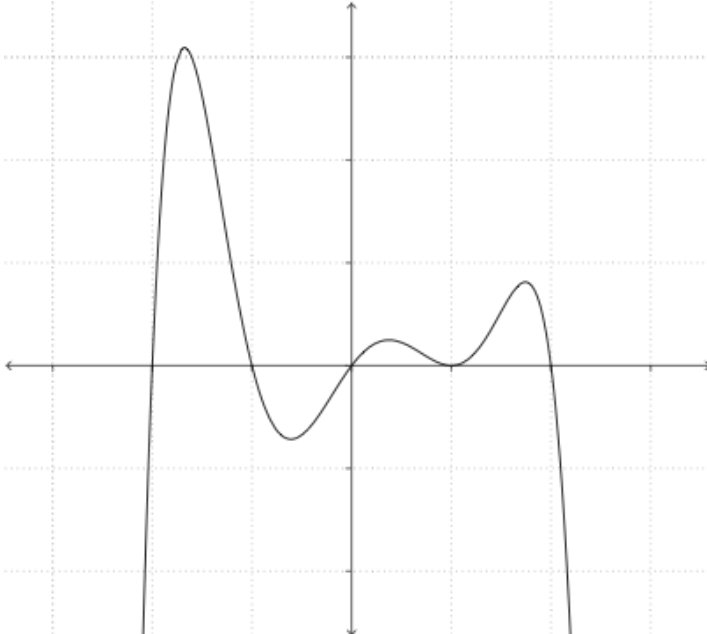
Hint: The picture below shows the names of the quadrants:



Question #8

Reference Q.11350

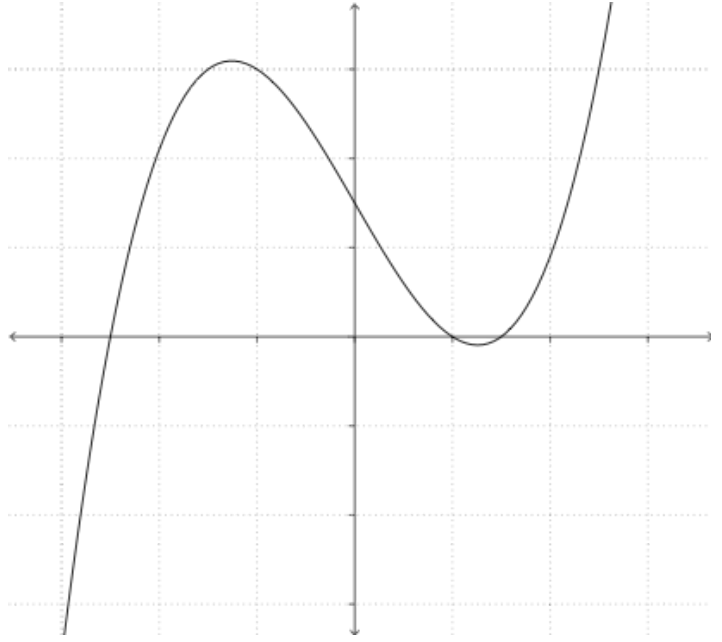
What is the minimum degree of the polynomial function sketched below?



Question #9

Reference Q.11348

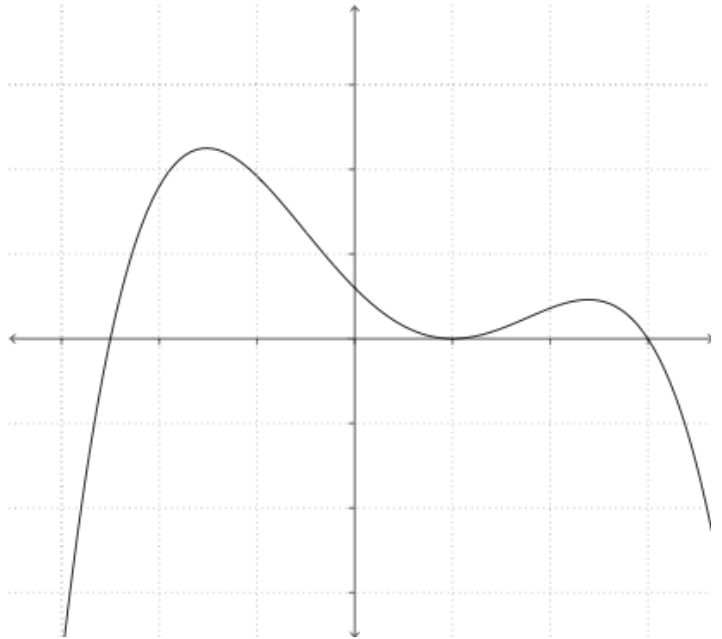
What is the minimum degree of the polynomial function sketched below?



Question #10

Reference Q.11349

What is the minimum degree of the polynomial function sketched below?



Question #11

Reference Q.11324

Sketch a simple graph of the following polynomial:

$$f(x) = (x - 3)(x + 2)$$

Question #12

Reference Q.11326

Sketch a simple graph of the following polynomial:

$$f(x) = -(x - 2)(x - 5)$$

Question #13

Reference Q.11328

Sketch a simple graph of the following polynomial:

$$f(x) = -\frac{1}{5}(x + 3)(x - 5)$$

Question #14

Reference Q.11329

Sketch a simple graph of the following polynomial:

$$f(x) = (x - 4)(x + 1)(x + 3)$$

Question #15

Reference Q.11330

Sketch a simple graph of the following polynomial:

$$f(x) = -(x + 2)(5x - 3)(x - 6)$$

Question #16

Reference Q.11331

Sketch a simple graph of the following polynomial:

$$f(x) = (x - 1)^2(x + 4)$$

Question #17

Reference Q.11333

Sketch a simple graph of the following polynomial:

$$f(x) = x(x + 4)(x - 5)(x - 7)$$

Question #18

Reference Q.12000

What is the value of the leading coefficient a if the polynomial function

$$P(x) = a(x - 2)(x - 6)(x + 1)$$
 passes through the point $(0, 3)$?

Question #19

Reference Q.12001

Consider a function $P(x)$ that passes through the points $(-3, 0)$, $(-2, 0)$, $(5, 0)$ and $(0, 20)$.

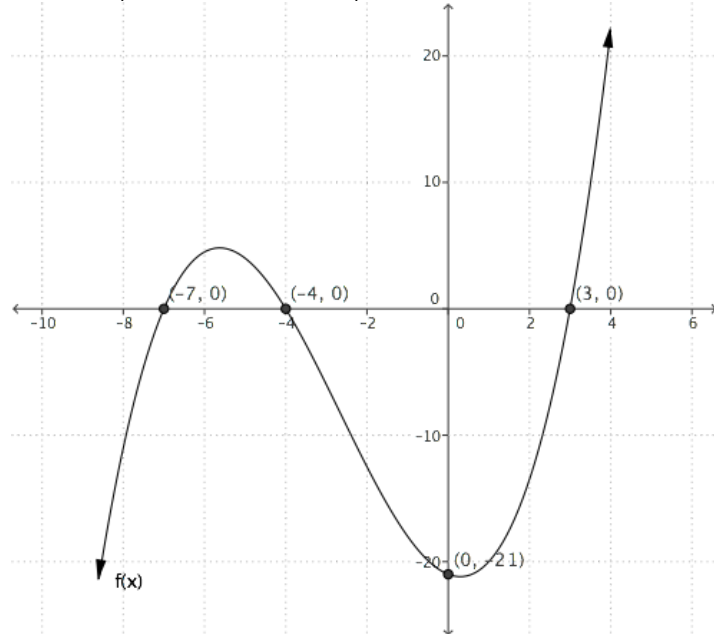
a. Write a function for $P(x)$.

b. Sketch $P(x)$.

Question #20

Reference Q.11352

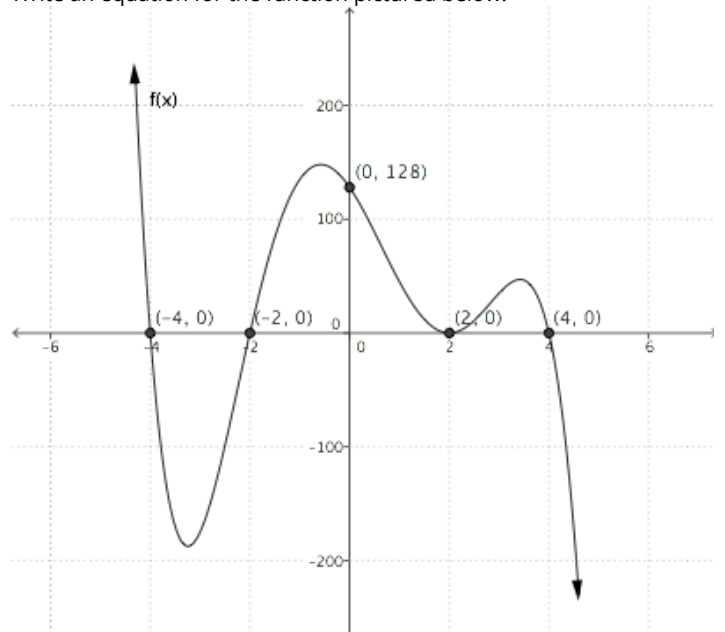
Write an equation for the function pictured below:



Question #21

Reference Q.11351

Write an equation for the function pictured below:



Question #22

Reference Q.12020

In each question use a graphing calculator to:

- sketch the graph of the polynomial function
- state the zeros of the polynomial function
- write the polynomial function in factored form

a. $P(x) = x - 2$

b. $P(x) = -x + 2$

c. $P(x) = x^2 - 6x + 8$

d. $P(x) = -x^2 + 6x - 8$

e. $P(x) = x^3 - 7x^2 + 7x + 15$

f. $P(x) = -x^3 + 7x^2 - 7x - 15$

g. $P(x) = x^3 - x^2 - 12x$

h. $P(x) = -x^3 + x^2 + 12x$

i. $P(x) = x^4 - 5x^2 + 4$

j. $P(x) = -x^4 + 5x^2 - 4$

Question #23

Reference Q.12021

In each question use a graphing calculator to:

- sketch the graph of the polynomial function
- state the zeros of the polynomial function
- write the polynomial function in factored form

a. $P(x) = x^4 - 7x^3 + 7x^2 + 15x$

b. $P(x) = -x^4 + 7x^3 - 7x^2 - 15x$

c. $P(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$

d. $P(x) = -x^5 + 3x^4 + 5x^3 - 15x^2 - 4x + 12$

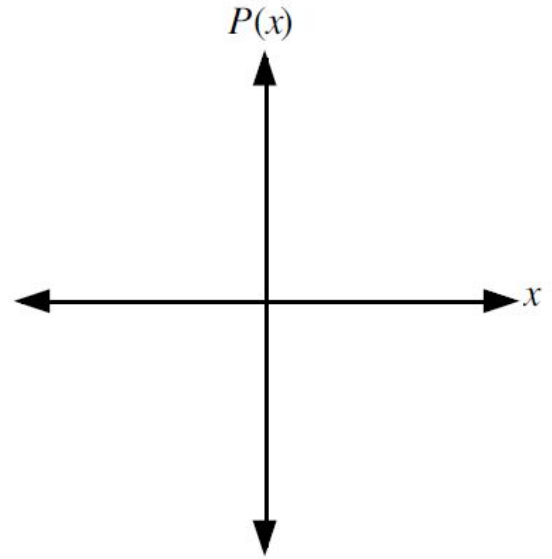
e. $P(x) = x^6 - 14x^4 + 49x^2 - 36$

f. $P(x) = -x^6 + 14x^4 - 49x^2 + 36$

Question #24

Reference Q.12031

- Choose appropriate windows which will enable you to investigate all the characteristics of the functions.
Graph $P(x) = x^3 - 4x^2 - 3x + 18$ and complete the table.



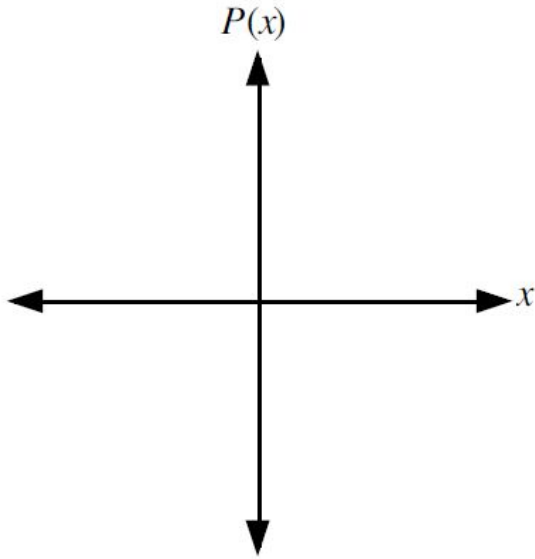
zero	multiplicity

- Write the polynomial in the form $P(x) = (x - a)(x - b)^2$, where $a, b \in I$.

🔗 **Question #25**

Reference Q.12032

- a. Choose appropriate windows which will enable you to investigate all the characteristics of the functions.
Graph $P(x) = x^4 + x^3 - 18x^2 - 52x - 40$ and complete the table.



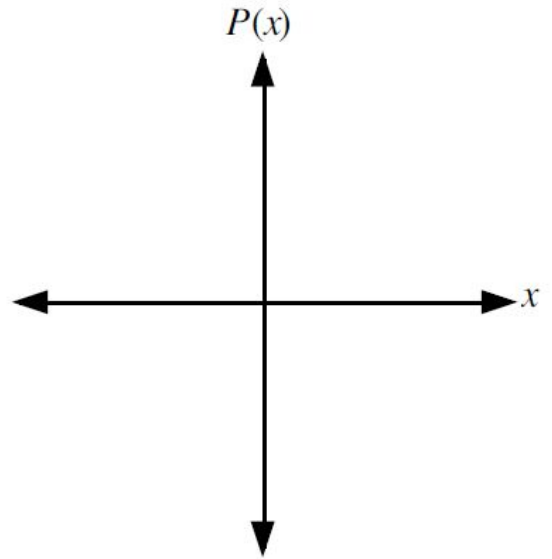
zero	multiplicity

- b. Write the polynomial function in factored form.

🔗 **Question #26**

Reference Q.12033

- a. Choose appropriate windows which will enable you to investigate all the characteristics of the functions.
Graph $P(x) = -x^3 - 6x^2 + 32$ and complete the table.



zero	multiplicity

- b. Write the polynomial in the form $P(x) = -(x - a)(x - b)^2$, $a, b \in I$

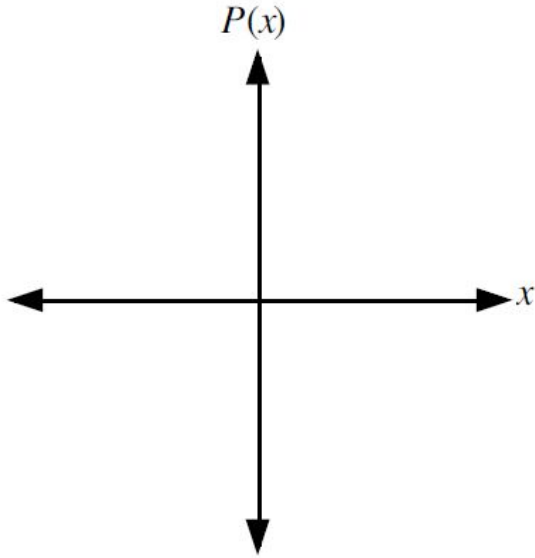
🔗 **Question #27**

Reference Q.12034

a. Sketch the graphs of

$$P(x) = x^4 - 4x^3 - 2x^2 + 12x + 9 \text{ and}$$

$$Q(x) = -x^4 + 4x^3 + 2x^2 - 12x - 9 \text{ on the grid.}$$



b. State the zeros, their multiplicities, and the **y**-intercept of each polynomial function.

c. Write the equations of the polynomials in factored form.

$$P(x) =$$

$$Q(x) =$$

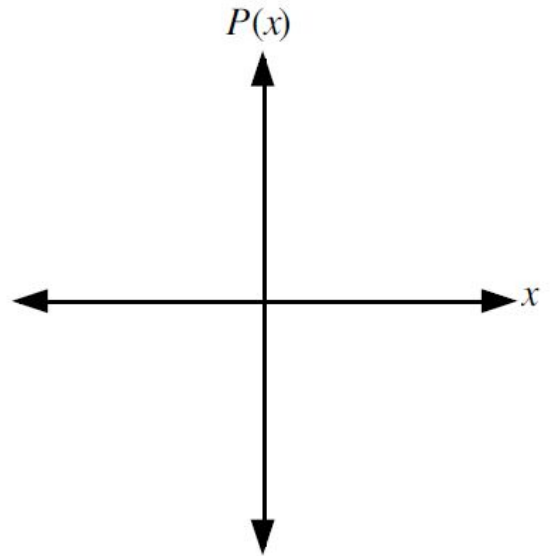
🔗 **Question #28**

Reference Q.12035

A cubic polynomial function has the equation

$P(x) = ax^3 + bx^2 + cx + d$ with a leading coefficient of 1. The zeros of the polynomial are **-6, 1, and 3**.

a. Sketch the graph of $P(x)$ and write the equation of the polynomial in factored form.



b. Determine the values of **a, b, c, and d** in $P(x)$

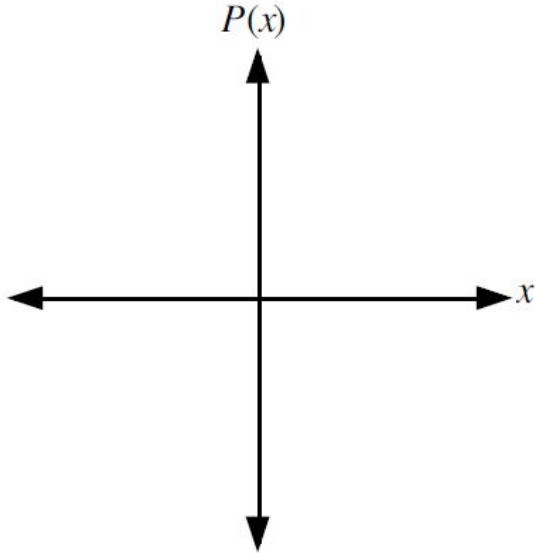
Question #29

Reference Q.12036

A cubic polynomial function has the equation

$P(x) = ax^3 + bx^2 + cx + d$ with a leading coefficient of 1. The function has two real distinct zeros. The zero 2 has multiplicity one and the zero -3 has multiplicity two.

- a. Sketch the graph of the function and write the equation of the polynomial in factored form.



- b. Determine the values of a , b , c , and d in $P(x)$.
- c. A new function is formed by changing the signs of each of the values of a , b , c , and d . Describe how the graph of the new function compares to the graph of $P(x)$.

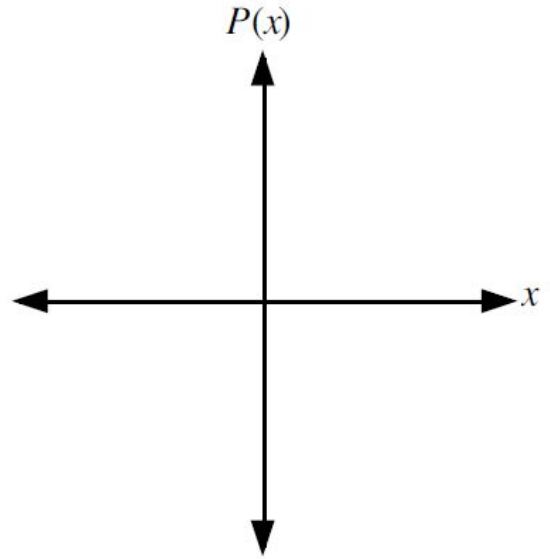
Question #30

Reference Q.12037

A polynomial function has the equation

$$P(x) = x^2(x - 2)(x + 3).$$

- a. Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.



- b. State the zeros, their multiplicities, and the y -intercept of $P(x)$.

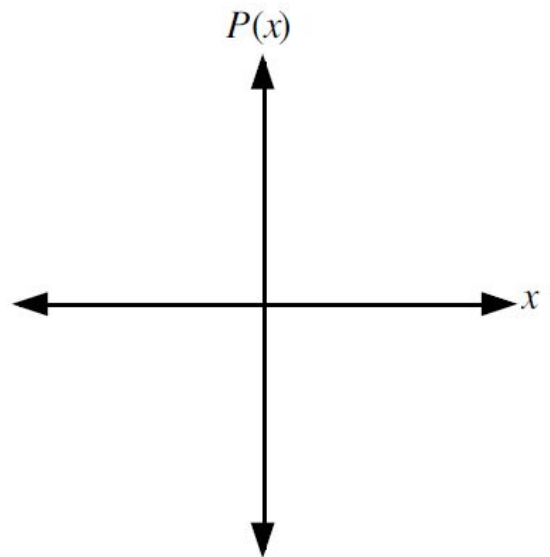
Question #31

Reference Q.12040

A polynomial function has the equation

$$P(x) = -(x - 4)^2(x + 3)^2.$$

- a. Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.



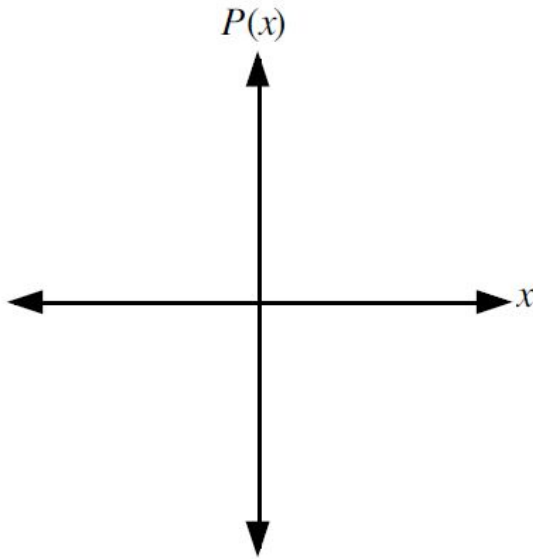
- b. State the zeros, their multiplicities, and the y -intercept of $P(x)$.

Question #32

Reference Q.12041

A polynomial function has the equation $P(x) = (x - 1)^3(x + 3)$.

- a. Make a rough sketch without using a graphing calculator.
Verify using a graphing calculator.



- b. State the zeros, their multiplicities, and the y -intercept of $P(x)$.

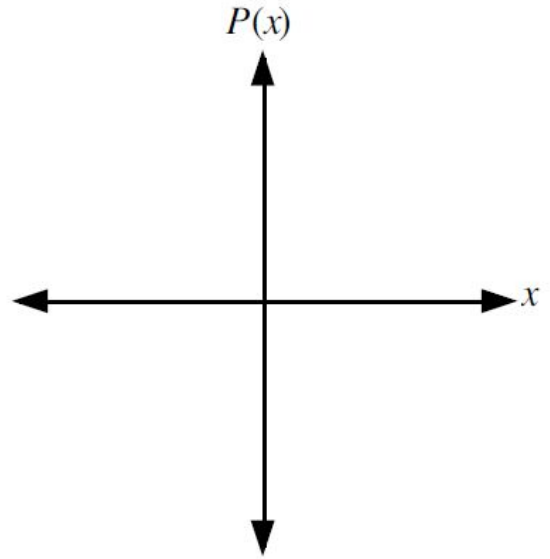
Question #33

Reference Q.12042

A polynomial function has the equation

$$P(x) = (x + 2)^2(x + 5)(3 - x).$$

- a. Make a rough sketch without using a graphing calculator.
Verify using a graphing calculator.



- b. State the zeros, their multiplicities, and the y -intercept of $P(x)$.
- c. Algebraically estimate the value of the maximum of $P(x)$.
(Hint: use your sketch to estimate the value of x that will maximize the $P(x)$.)

Question #34

Reference Q.12075

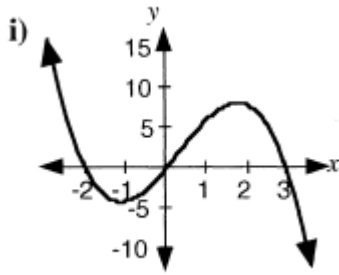
Complete the following based on your observations from questions #1 - #10.

- a. The polynomial function $P(x) = (x - a)(x - b)^2(x - c)^3$ has a zero at $x = a$, then the graph of the function at $x = a$ _____.
- b. The polynomial function $P(x) = (x - a)(x - b)^2(x - c)^3$ has a zero at $x = b$, then the graph of the function at $x = b$ _____.
- c. The polynomial function $P(x) = (x - a)(x - b)^2(x - c)^3$ has a zero at $x = c$, then the graph of the function at $x = c$ _____.
- d. The polynomial function $P(x) = (x - a)(x - b)^2(x - c)^3$ has three distinct zeros.
If $a < b < c$, make a rough sketch of a polynomial which satisfies these conditions.

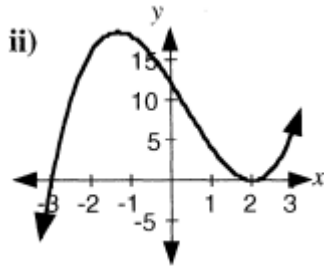
Question #35

Reference Q.12076

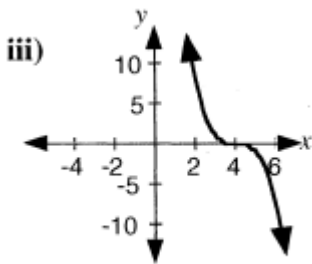
The graphs shown each represent a cubic polynomial function with equation $P(x) = ax^3 + bx^2 + cx + d$, where a is 1 or -1 . The x -intercepts on the graphs are integers. In each case, write $P(x)$ in factored form and determine the values of $a, b, c,$ and d .



a.



b.



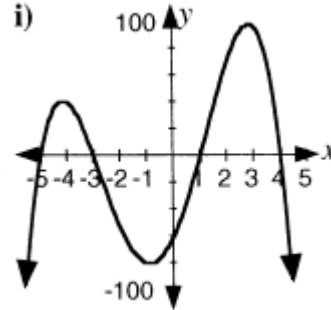
c.

Question #36

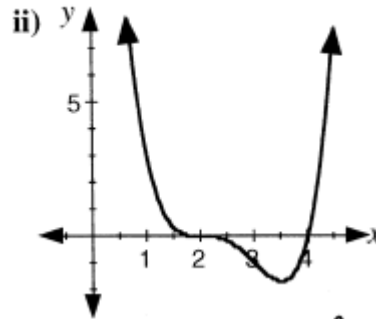
Reference Q.12077

The graphs shown below each represent a quartic polynomial function with equation $P(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a is 1 or -1 . The zeros of the functions are integers.

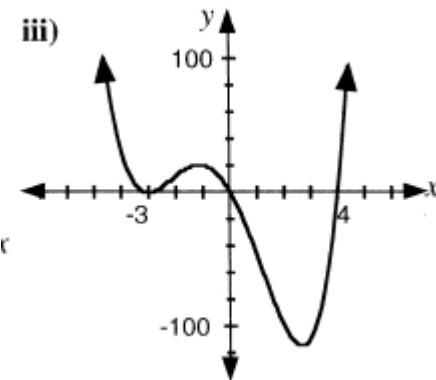
In each case, write the equation of the polynomial function in factored form and determine the value of e .



a.



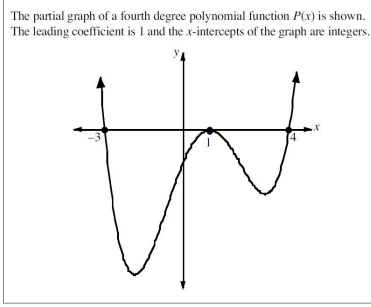
b.



c.

Question #37

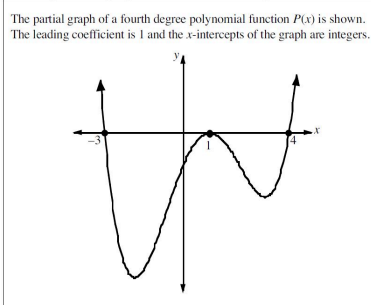
Reference Q.12078



If the polynomial function is written in the form $P(x) = c(x - a)^2(x - b)(x + d)$, where a, b, c , and d are all positive integers, then the respective numerical values of a, b, c, d are _____.

Question #38

Reference Q.12079



The graph crosses the y -axis at $(0, -m)$. The value of m is _____.

Question #39

Reference Q.12038

How does the concept **number of zeros** differ from the concept **multiplicity of zeros**?

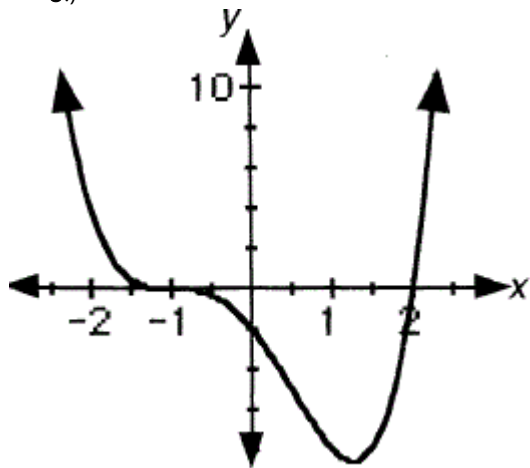
Question #40

Reference Q.12039

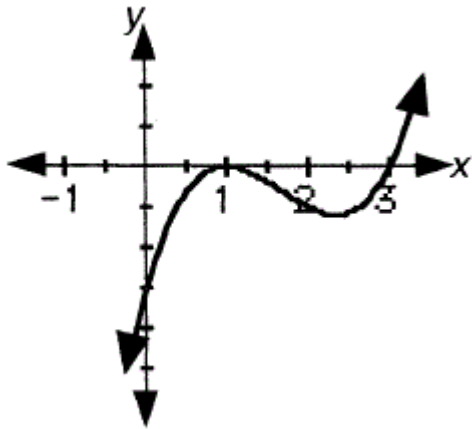
Consider the graphs below.

a. In each case state the number of distinct zeros and the possible multiplicities of each zero.

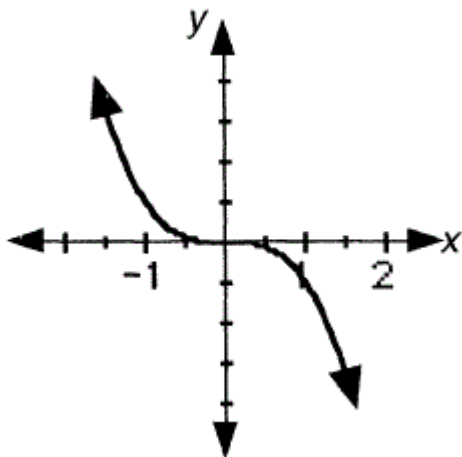
(Multiplicity refers to the number of times a zero appears in a polynomial. For instance $P(x) = (x + 2)(x + 3)^2$ has a zero of multiplicity of 1 at $x = -2$ and a zero of multiplicity of 2 at $x = -3$.)



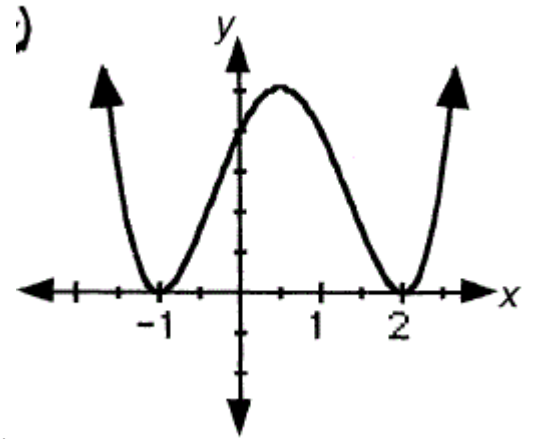
ii.



iii.



iv.



v.

b. Which graph(s) could represent a seventh-degree polynomial function?

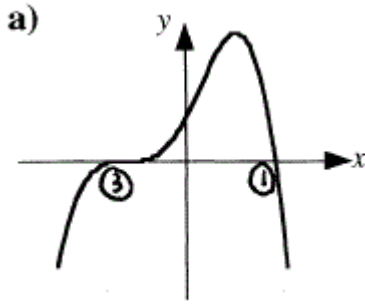
c. Which graph(s) could **not** represent a polynomial function of degree 10?

d. In which of the graphs is the leading coefficient negative?

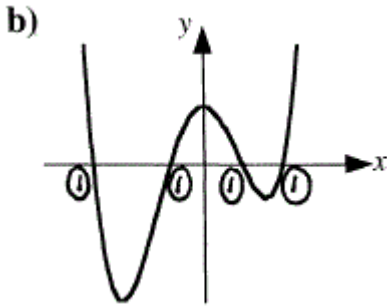
Question #41

Reference Q.12113

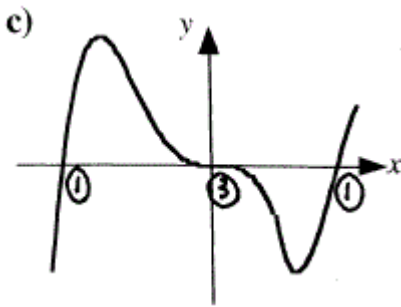
The following graphs represent functions of lowest possible degree. State the degree in each case.



a.



b.

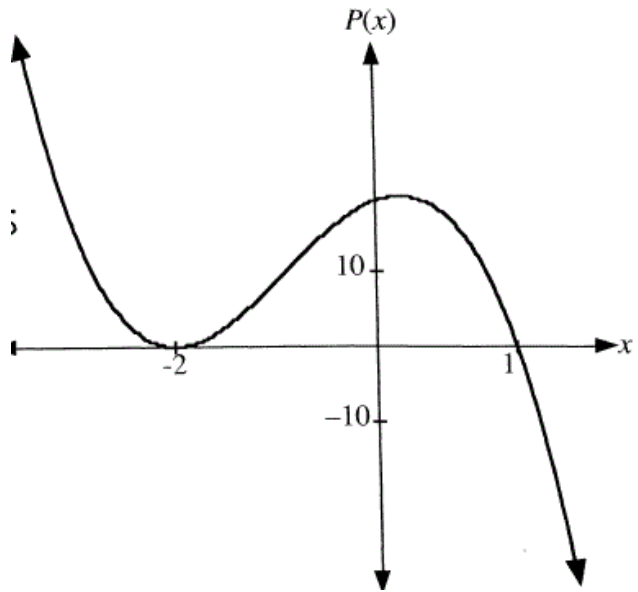


c.

Question #42

Reference Q.12114

The graph represents a polynomial function $P(x)$ of degree 5, and use the fact that $P(0) = 16$.



- Write the equation of $P(x)$ in factored form if the leading coefficient is -1 . Assume the roots of $P(x)$ are all real.
- Estimate, to the nearest whole number, the maximum value of $P(x)$ on the domain $-2 \leq x \leq 1$.
- (Optional: Challenge Problem)**
Prove the same result in part (a) without assuming that the roots of $P(x)$ are all real.
Write the equation of $P(x)$ in factored form if the leading coefficient is -1 .

Question #43

Reference Q.12164

- On the grid, sketch a graph of a polynomial function satisfying the given conditions.
 - negative leading coefficient
one real zero of multiplicity 1
one real zero of multiplicity 2
two real zeros of multiplicity 3
(many answers are possible)
 - positive leading coefficient
two real zeros of multiplicity 2
one real zero of multiplicity 3
one real zero of multiplicity 6
(many answers are possible)
- State the degree of each polynomial function.
 -
 -
 -
 -

Question #44

Reference Q.12160

The cubic function $f(x) = x^3 - 12x + 65$ has one real zero of multiplicity 1 and two non-real zeros.

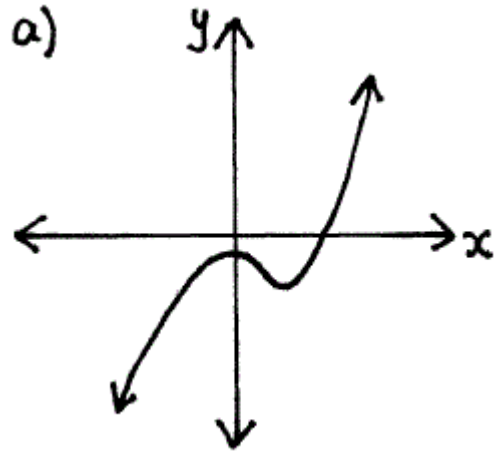
- Use a graphing calculator to graph the function and make a sketch of the graph on the grid.
- Use synthetic division to determine the real zero of the function.
- Determine the two non-real zeros of the function.

Question #45

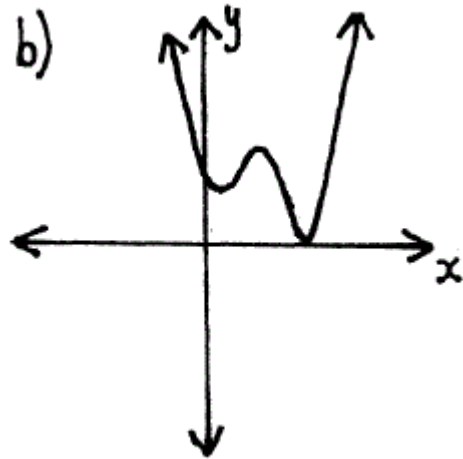
Reference Q.12161

In each case, sketch the graph of the polynomial function using a graphing calculator window of $x : [-8, 8, 1]$ and $y : [-50, 50, 10]$ and write a statement which describes the number, type, and multiplicity of the zeros.

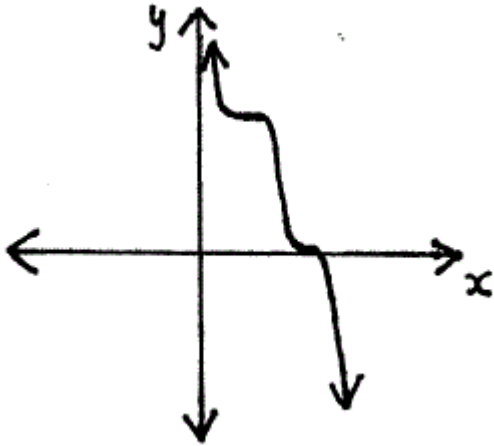
a. $P(x) = x^3 - 6x^2 + 6x - 5$



b. $Q(x) = x^4 - 11x^3 + 36x^2 - 35x + 25$

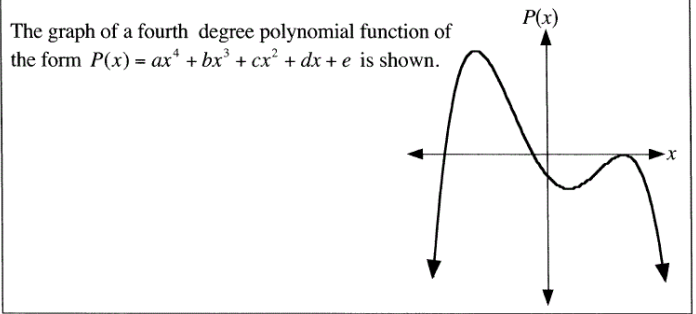


c. $R(x) = -x^5 + 13x^4 - 61x^3 + 124x^2 - 112x + 64$



Question #47

Reference Q.12163

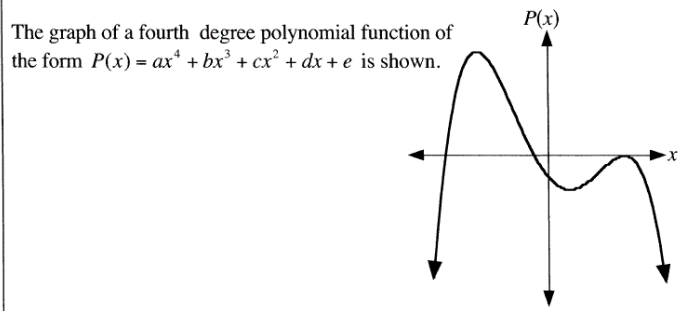


If $P(x) = 0$ has exactly three different solutions, then which one of the following statements about the roots of $P(x) = 0$ is true.

- A. Two roots are real, equal and negative, and two roots are real, not equal and positive.
- B. Two roots are real, equal and positive, and two roots are real, not equal and negative.
- C. Two roots are real, equal and negative, and two roots are not real.
- D. Two roots are real, equal and positive, and two roots are not real.

Question #46

Reference Q.12162



The value a and e must satisfy

- A. $a > 0, e < 0$
- B. $a < 0, e > 0$
- C. $a > 0, e > 0$
- D. $a < 0, e < 0$