

AP Calculus AB/BC - Applications of Integration

Show all your work. Clearly indicate the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations. Use complete sentences for all explanations.

1. Zahra and Miguel race their rabbits, Zanny and Manny, along a straight track for 4 minutes. The rabbits' velocity functions are given below in metres per minute.

Zanny	$Z(t) = \frac{120}{t^2 - 2t + 9}$
Manny	$M(t) = -t \ln(t) + t + 12$

- a) Calculate Zanny's average velocity over 4 minutes (remember to include proper units in your final answer).
- b) Over the course of the 4 minutes, what is the largest distance between the two rabbits?
- c) At the beginning of the race, both Zanny and Manny are accelerating. Are they still accelerating at the end of the race? If not, find the values of t when they each switch from accelerating to decelerating. Remember to state your final answer using complete sentences.

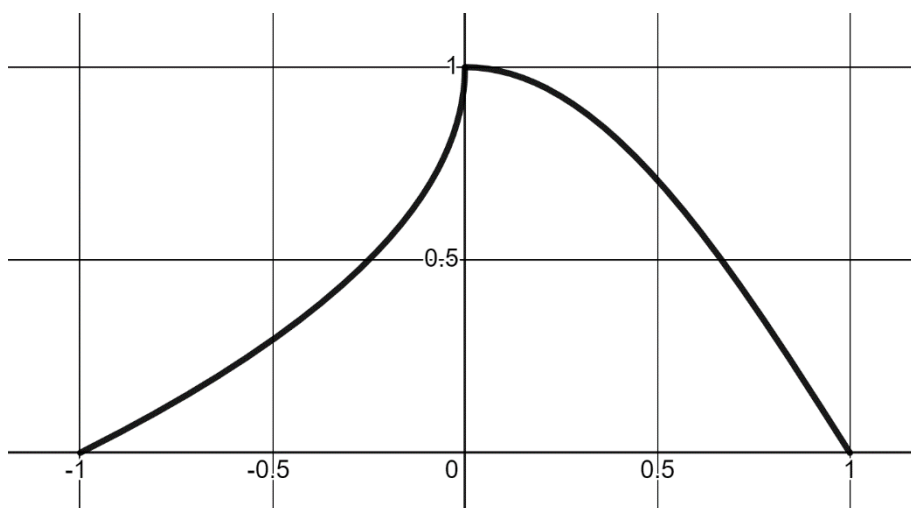
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d) When Manny starts decelerating, is the distance between the two rabbits increasing or decreasing?

e) Which rabbit wins the race? Explain your conclusion.

2. The function $f: [-1, 1] \rightarrow \mathbb{R}$ is defined piecewise and graphed below.

$$f(x) = \begin{cases} -\sqrt{-x} + 1, & -1 \leq x < 0 \\ \cos\left(\frac{\pi x}{2}\right), & 0 \leq x \leq 1 \end{cases}$$



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Let R be the region of the x - y plane bounded by the function $f(x)$ and the x -axis.

a) Calculate the area of R .

b) R is the base of a solid where the cross sections perpendicular to the x -axis are equilateral triangles. Calculate the volume of the solid.

c) Consider the region of the x - y plane bounded by $f(x)$ and the line $y = \frac{1}{2}$. Rotate that region around the x -axis to form a solid. Calculate the volume of the solid.

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3. The rate at which your home consumes electricity is measured in kilowatts. If your home consumes electricity at the rate of 1 kilowatt for 1 hour, you will be charged for 1 kilowatt-hour of electricity. Suppose that the average consumption rate for a certain home is modeled by the function

$$C(t) = 3.9 - 2.4 \sin \frac{\pi t}{12}$$

where $C(t)$ is measured in kilowatts and t is the number of hours past midnight. Find the average daily consumption for this home, measured in kilowatt-hours.

4. Population density measures the number of people per square mile inhabiting a given living area. Washerton's population density, which decreases as you move away from the city center, can be approximated by the function $10000(2 - r)$ at a distance r miles from the city center.
- a) If the population density approaches zero at the edge of the city, what is the city's radius?

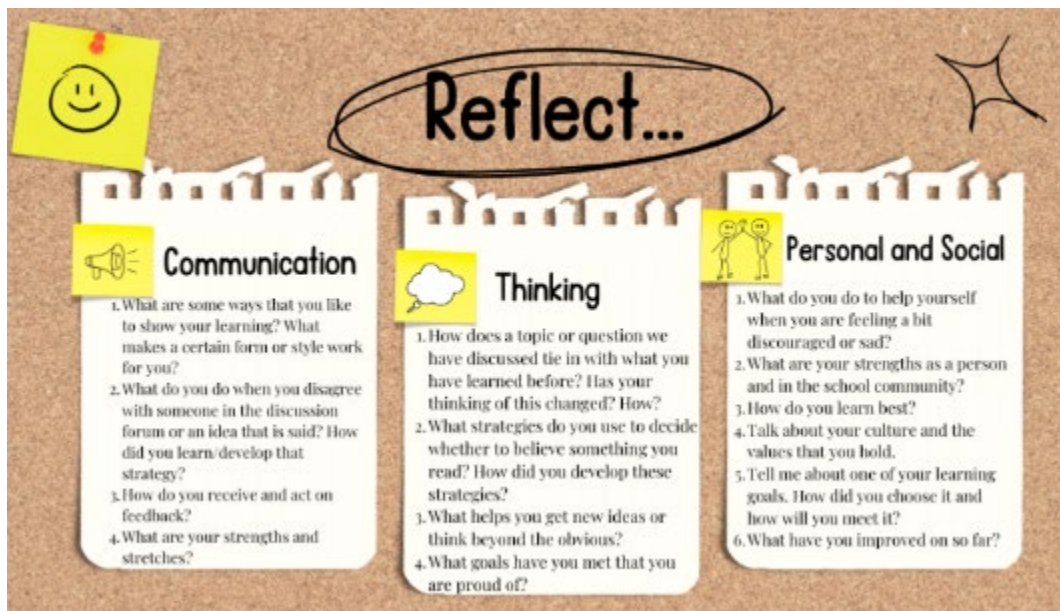
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- b) A thin ring around the center of the city has thickness Δr and radius r . If you straighten it out, it is a rectangular strip. Approximately what is its area?

- c) Estimate the total population of Washerton by setting up and evaluating a definite integral.

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5. Think about your learning journey in this unit. Pick a category and write a response to ONE of the category's questions in a few complete sentences.



~The End~

Quick checklist:

Did you...

- ☐ State your final answer and explanations using full sentences?
- ☐ Explain in a few sentences every time you have a Yes/No answer?
- ☐ Use equal signs correctly where necessary?