Lesson 8: Applications of Trigonometric Functions

Question #1

Reference Q.12475

The alarm in a noisy factory is a siren whose volume, V decibels, fluctuates so that t seconds after starting, the volume is given by the function $V(t) = 18\sin\frac{\pi}{15}t + 60.$

a. What are the maximum and minimum volumes of the siren?

b. Determine the period of the function.

c. Write a suitable window (on a graphing calculator) which can be used to display the graph of the function.

d. After how many seconds, to the nearest tenth, does the volume first reach 70 decibels?

e. The background noise level in the factory is 45 decibels. Between which times, to the nearest tenth of a second, in the first cycle is the alarm siren at a lower level than the background noise?

f. For what percentage, to the nearest percent, of each cycle is the alarm siren audible over the background factory noise?

Question #2

Reference Q.12476

A top secret satellite is launched into orbit from a remote island not on the equator. When the satellite reaches orbit, it follows a sinusoidal pattern that takes it north and south of the equator (i.e. the equator is used as the horizontal axis). Twelve minutes after it is launched it reaches the farthest point north of the equator. The distance north or south of the equator can be represented by the function $d(t) = 5000\cos\left[\frac{\pi}{35}(t-12)\right]$ where $d(t)$ is the distance, in km, of the satellite north of the equator t minutes after being launched.

a. How far north or south of the equator is the launch site? Answer to the nearest km.

b. Is the satellite north or south of the equator after 20 minutes? What is this distance to the nearest kilometre?

c. When, to the nearest tenth of a minute, will the satellite first be 2500 km south of the equator?

Question #3

Reference Q.12490

The height of a tidal wave approaching the face of the cliff on an island is represented by the equation $h(t) = 7.5\cos\left(\frac{2\pi}{9.5}t\right)$ where $h(t)$ is the height, in metres, of the wave above normal sea level t minutes after the wave strikes the cliff.

a. What are the maximum and minimum heights of the wave relative to normal sea level?

b. What is the period of the function?

c. How high, to the nearest tenth of a metre, will the wave be, relative to normal sea level, one minute after striking the cliff?

d. Normal sea level is 6 metres at the base of the cliff.
   v. For what values of h would the sea bed be exposed?
   vi. How long, to the nearest tenth of a minute, after the wave strikes it highest mark on the cliff does it take for the sea bed to be exposed?
   vii. For how long, to the nearest tenth of a minute, is the sea bed exposed?

Question #4

Reference Q.12491

The depth of a water in a harbour can be modelled by the function $d(t) = -5\cos\frac{\pi}{6}t + 16.4$ where $d(t)$ is the depth in metres and t is the time in hours after low tide.

a. What is the period of the tide?

b. A large cruise ship needs at least 14 metres of water to dock safely. For how many hours per cycle, to the nearest tenth of an hour, can a cruise ship dock safely?

Question #5

Reference Q.12492

A city water authority determined that, under normal conditions, the approximate amount of water, $W(t)$, in millions of litres, stored in a reservoir t months after May 1, 2012, is given by the formula $W(t) = 1.25 - \sin\frac{\pi}{6}t.$

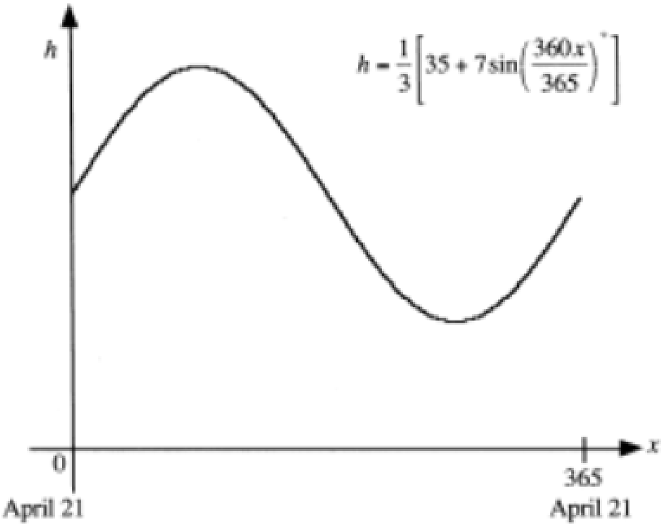
a. Sketch the graph of this function over the next three years.

b. In the summer of 2012, the authority decided to carry out the following simulation to determine if they had enough water to cope with a serious fire. "If, on November 1, 2013, there is a serious fire which requires 300000 litres of water to be brought under control, will the reservoir run dry if water rationing is not imposed?"
   iii. Explain how to use the graph in a) to solve the problem.
   iv. Will the reservoir run dry if water rationing is not imposed? If so, in what month will this occur?

Question #6

Reference Q.12493

The graph shows how the number of hours (h) of daylight in a European city changes during the year.

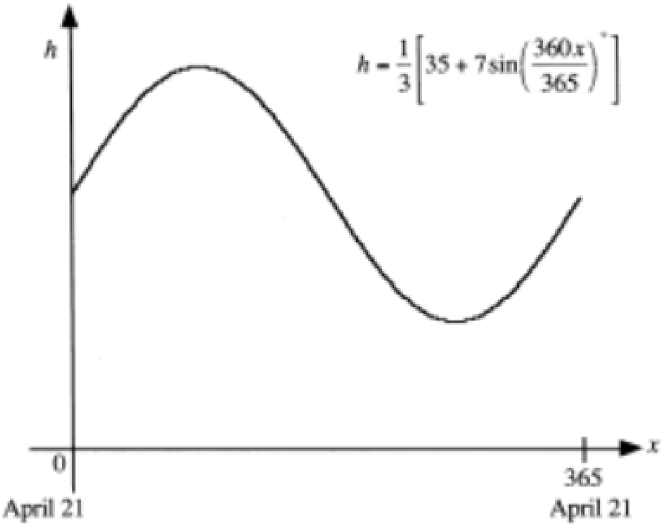


Mid-winter is the day with the least hours of daylight. The number of hours of daylight, to the nearest tenth of an hour, that there will be on mid-winter's day is ____.

Question #7

Reference Q.12494

The graph shows how the number of hours (h) of daylight in a European city changes during the year.

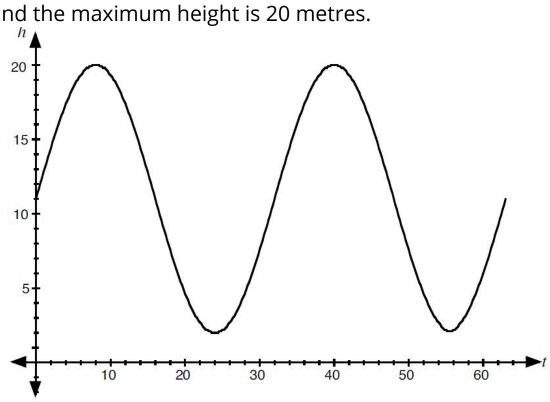


The number of days after April 21 untl mid-winter occurs is ____.

Question #8

Reference Q.12488

The graph shows the height, h metres, above the ground over time, t, in seconds that it takes a person in a chair on a Ferris Wheel to complete two revolutions. The minimum height of the Ferris Wheel is 2 metres and the maximum height is 20 metres.



a. How far above the ground is the person as the wheel starts rotating?

b. If it takes 16 seconds for the person to return to the same height, determine the equation of the graph in the form $h(t) = a\sin bt + d$.

c. Determine the distance the person is from the ground, to the nearest tenth of a metre, after 30 seconds.

d. How long from the start of the ride does it take for the person to be at a height of 5 metres? Answer to the nearest tenth of a second.

Question #9

Reference Q.12489

A Ferris Wheel ride can be represented by a sinusoidal function. A Ferris Wheel at Westworld Theme Park has a radius of 15 m and travels at a rate of six revolutions per minute in a clockwise rotation. Ling and Lucy board the ride at the bottom chair from a platform one metre above the ground.

a. Sketch three cycles of a sinusoidal graph to represent the height Ling and Lucy are above the ground, in metres, as a function of time, in seconds.

b. Determine the equation of the graph in the form $h(t) = a\cos[b(t-c)] + d$.

c. If the Ferris Wheel does not stop, determine the height Ling and Lucy are above the ground after 28 seconds. Give answer to the nearest tenth of metre.

d. How long after the wheel starts rotating do Ling and Lucy first reach 12 metres from the ground? Give answer to the nearest tenth of a second.

e. How long does it take from the first time Ling and Lucy reach 12 metres until they next reach 12 metres from the ground? Give answer to the nearest second.

Question #10

Reference Q.12555

Consider the following information for a town in Saskatchewan for a leap year of 366 days:

- The latest sunrise time is at 09:00 on December 21 (day 356).
- The earliest sunrise time at 03:30 on June 21 (day 173).
- There is NO daylight saving time in Sasketchewan.
- The sunrise times vary sinusoidally with the day of the year.

a. Write a sinusoidal equation which relates the time of sunrise, $t,$ to the day of the year $d.$

b. Use the equation to determine what time, to the nearest minute, the sun rises on March 11.

c. Determine the average time the sun rises throughout the year.

d. How many days of the year does the sun rise before 6 a.m.?

Question #11

Reference Q.12557

Use the following information to answer the next question.

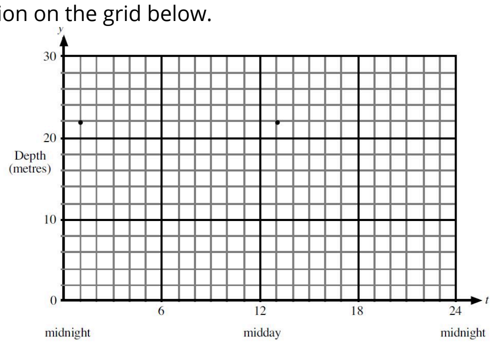
In Inverdeen harbour, the maximum dept of water is 22 metres at 1 a.m. and 1 p.m. as shown on the grid below.

The minimum depth of water is 6 metres at 7 a.m. and 7 p.m.

The depth is 14 metres at 4 a.m., 10 a.m., 4 p.m. and 10 p.m.

Assume that the relation between the depth of water, y metres, and the time, t hours, is a sinusoidal function.

a. If $t = 0$ at midnight, sketch the graph of the sinusoidal function on the grid below.



b. State the amplitude and period of the sinusoidal function. (Include units in the answers.)

c. Determine an equation of the sinusoidal function in the form $y = a\sin[b(t-c)] + d,$ where $a > 0$ and $c > 0.$

d. If the equation of the sinusoidal function is written in the form $y = a\cos[b(t-c)] + d,$ where $a > 0$ and $c > 0,$ only one of the parameters, a, b, c, d will be different from the values in c).

State which parameter will be different and give its value.

e. Calculate the depth of the water, to the nearest tenth of a metre, at 3:30pm.

Question #12

Reference Q.12559

Andrea, a local gymnast, is doing timed bounces on a trampoline. The trampoline mat is 1 metre above ground level. When she bounces up, her feet reach a height of 3 metres above the mat, and when she bounces down her feet depress the mat by 0.5 metres. Once Andrea is in a rhythm, her coach uses a stopwatch to make the following readings:

At the highest point the reading is 0.5 seconds.
At the lowest point the reading is 1.5 seconds.

a. Determine the maximum and minimum heights of Andrea's feet above the ground as she is bouncing on the trampoline.

b. Sketch two periods of the graph of the sinusoidal function which represents Andrea's height above the ground, in metres, as a function of time, in seconds.

c. How high was Andrea above the mat when the coach started timing?

d. Determine the equation of the graph in the form $h(t) = a\sin bt + d.$

e. How high, to the nearest tenth of a metre, was Andrea above the ground after 2.7 seconds?

f. Determine Andrea's exact height above the mat after 17 seconds.

g. How long after the timing started did Andrea first touch the mat?
Answer to the nearest tenth of a second.