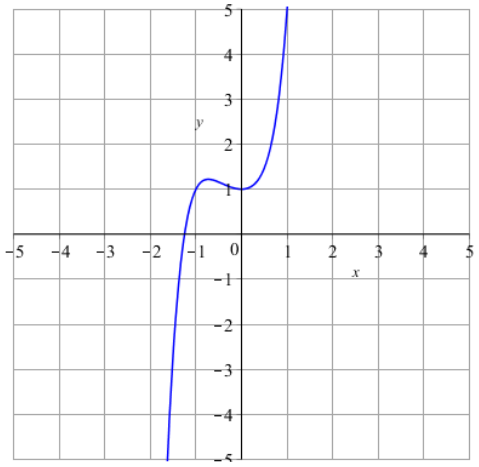
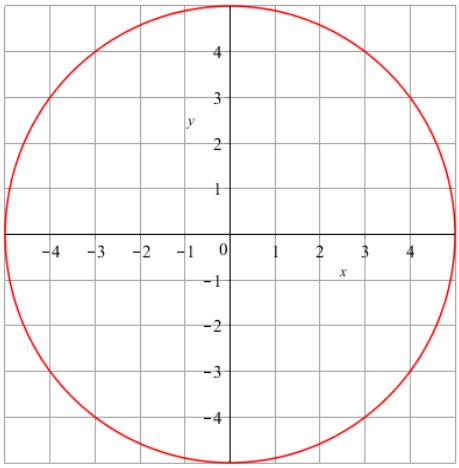
FUNCTION OPERATIONS AND CONICS

Name:	
Date:	
My Achievement Goal for this chapter is… (Circle one) A+ A B C D	
This Assignment is:	
Good to Go	Needs Corrections

Please answer all questions on this sheet and remember to show all your work!

1. Consider the two graphs given below:





   a. Which of the graphs represents a function? (Left one / Right one)

   b. Explain why the other graph is not a function.

   c. Find the domain and the range of both graphs.

   d. Which of the inverse functions of each graph is also a function?
   (Hint: You do not need to sketch their inverses. Think of the shortcut to check whether a graph is a function.)

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2. Let $f(x) = 4x^2$ and $g(x) = x^{-1}$. Simplify or evaluate the following expressions:
   a. $f(x) + g(x)$

   b. $g(x) - f(x)$

   c. $f(x) \cdot g(x)$

   d. $\frac{g(x)}{f(x)}$

   e. $g(f(x))$

   f. $g(f(x-2))$

   g. $\sqrt{f(f(x-1))}$

   h. $g(g(2))$

   i. Solve the expression $f\left(\frac{1}{2}x\right) = g\left(\frac{1}{x}\right)$ for x.

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3. Let $f(x) = |x - 1|$ and $g(x) = x^2 + 2$.
   a. Find $(fg)(-2)$.

   b. Find the value of $g\left(f\left(-1 + \sqrt{2}\right)\right)$.

4. Let $f: \{0, 1, 2\} \to \mathbb{R}$ and $g: \{0, 1, 2\} \to \mathbb{R}$ be two functions over $\mathbb{R}$. If $f(0) = 0, f(1) = -2$, $f(2) = 4$, and $g(0) = 3, g(1) = 2, g(2) = -8$, answer the following:
   a. What are the domain and range of both f and g?

$$\text{Domain}(f) = \{ \quad \}$$
$$\text{Range}(f) = \{ \quad \}$$
$$\text{Domain}(g) = \{ \quad \}$$
$$\text{Range}(g) = \{ \quad \}$$

   b. $|(f + g)(2)|$

   c. $(fg)(1)$

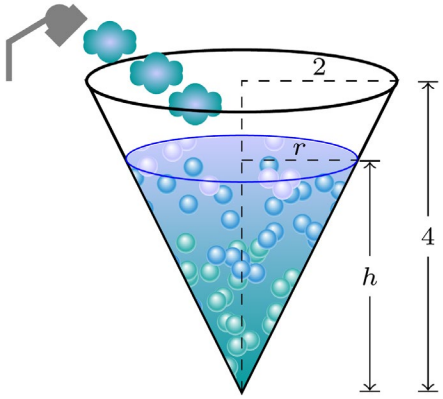
   d. $\left(\frac{f}{g}\right)(0)$

   e. $\left(\frac{f}{g^{-1}}\right)(2)$

5. A water tank has the shape of an inverted circular cone with base radius $2m$ and height $4m$. If water is being pumped into the tank, and at certain time $t_0 \neq 0$, the volume of the water V is given by

$$V = \frac{1}{3}\pi r^2 h,$$

where r is the radius of the surface and h is the height of the water at a specific time during the pumping process.



It is very useful to express V as a function of h, $V(h)$ alone.

a. Find the expression $V(h)$.

(Hint: Think of similar triangles)

b. Find the volume when $h = 2m$

c. Briefly, sketch the behaviour of the height h with respect to $t \geq 0$.

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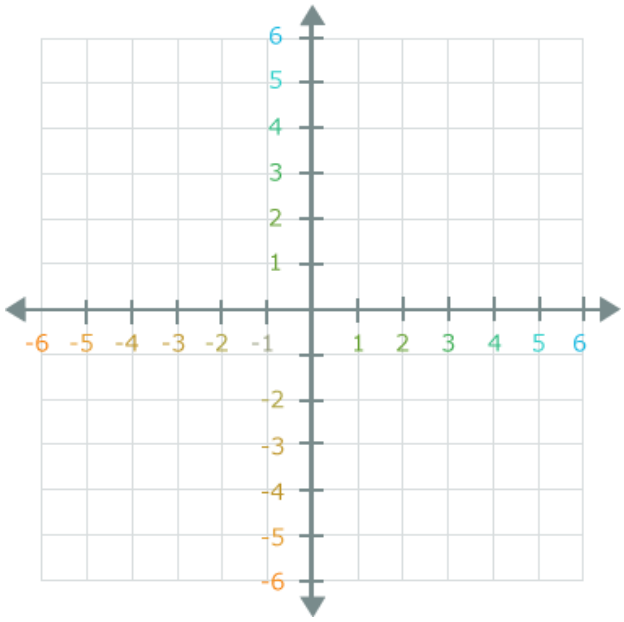
6. Consider $f(x) = (x-1)^2 + 2$.
   a. State the domain and range of $f(x)$.

   b. Find the vertex $V = (p, q)$ of $f(x)$. What is the image of V if $f(x)$ is reflected through the line $y = x$?

   c. Sketch $y = f^{-1}(x)$.



   d. Find the inverse of $f(x)$ algebraically. Confirm your answer by computing $f(f^{-1}(x))$ and $f^{-1}(f(x))$.

   e. Restrict the domain of $f(x)$ so that $f^{-1}(x)$ is also a function.

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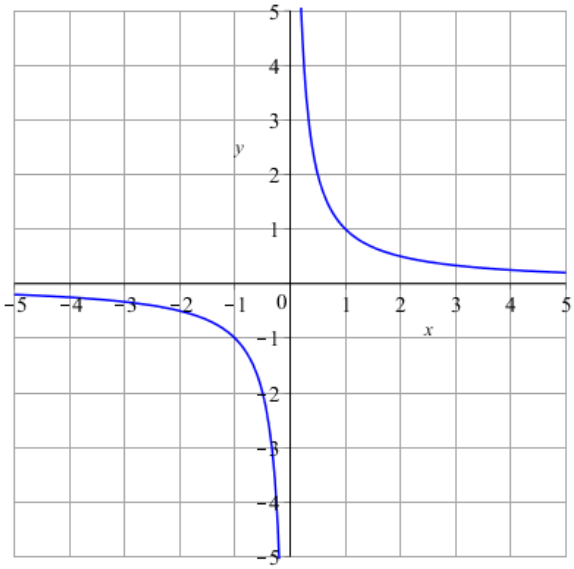
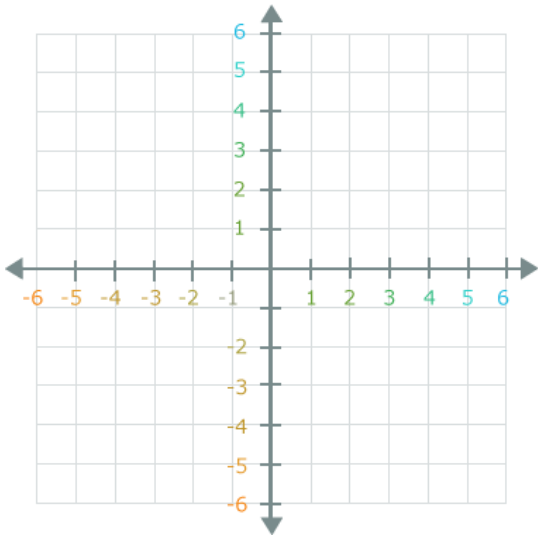
7. The graph of $y = f(x)$ is shown. sketch the graph of $x = -f(y)$ on the same grid provided.



8. Consider the equation $(x-2)^2 + (y-3)^2 = 9$
   a. Sketch the conic



   b. If there is a vertical stretch by a factor of 2 determine the equation of the conic in standard form.

9. Graph the following equation and identify the important characteristics of the graph such as center, vertices, foci, and asymptotes.

$$\frac{(x-2)^2}{4} - \frac{(y-2)^2}{5} = 1$$

Center:

Vertices:

Foci:

Asymptotes: