# TRIGONOMETRIC EQUATIONS AND IDENTITIES

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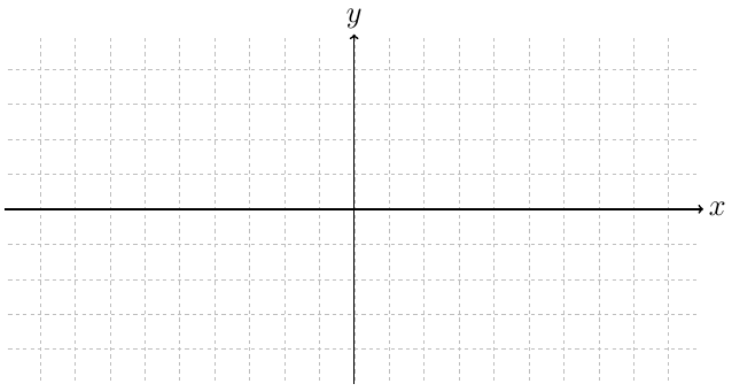
| **Name:** | |
| **Date:** | |
| My Achievement Goal for this chapter is… (Circle one)<br>A+ A B C D | |
| **This Assignment is:** | |
| Good to Go | Needs Corrections |

**Please answer all questions on this sheet and remember to show all your work!**

1. Let $y_1(x) = \cos x$ and $y_2(x) = \frac{1}{2}$.
   a. Sketch $y_1(x)$ and $y_2(x)$ on the same grid provided:




   b. How many solutions are there for $\cos x = \frac{1}{2}$ where $0 \leq x \leq 2\pi$?

   c. How many solutions are there for $\cos(2x) = \frac{1}{2}$ where $0 \leq x \leq 2\pi$?

2. Solve the equation $2\csc 2x + \sqrt{2} = 0$ for $0 \leq x \leq 2\pi$.

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3. Solve $\tan\left(x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$ for all $x \in \mathbb{R}$.

4. Solve the equation $\sin(2x - 0.35) = \cos(3x)$ where $0 \leq x \leq \pi$.

5. Recall that a function is periodic of period $T$ if $f(x + T) = f(x)$ for all $x$ in the domain of $f$. The smallest positive value of $T$ is called the fundamental period. Given that
$$f(x) = \pi x^2 - \cos(x).$$
Show that $f(x)$ is not a periodic function.
(Hint: Assume $f(x)$ is a periodic function of period $T$, where $T > 0$. Then, $f(x + T) = f(x)$ for all $x$ in the domain of $f$. Now, try to obtian a contradiction to the given assumption.)

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6. Solve $\cos(2\theta) = \sin(\theta)$ algebraically where $\theta \in \mathbb{R}$.

7. For what values of $x$ is the expression

$$\frac{\tan(x)}{1 - 2\cos^2 x}$$

defined?

8. State the Pythagorean, Reciprocal, Sum and Difference, and Double Angle identities.

9. Find the exact value of $\cos(15°)$ without using any calculator.
   (Hint: Use one of sum and difference identities)

10. Consider the following equations:
    I. $\sin(2u) = 2\sin u$
    II. $\sin(2u) = \frac{\sin u}{2}$
    III. $\frac{1}{2}\sin(2u) = \sin(u)\cos(u)$
    IV. $2\sin(2u) = \sin(u)\cos(u)$

    a. Do you think any of the above equations is an identity? Why do you think that?

    b. Prove any of the equations that are identities.

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11. Prove the following identities:
    a. $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

    b. $2 \sin\theta \cos^3\theta + 2 \sin^3\theta \cos\theta = \sin 2\theta$

    c. $\csc x - \cos x \cot x = \sin x$