Chapter 2: Quadratic Functions

Lesson 4: Maximum and Minimum Problems

Question #1

Reference Q.45518

A stunt woman on a motorbike runs up a ramp and launches into the air, taking a path according to the following equation: $\mathbf{f(x) = -5x^2 + 30x + 35}$, where $\mathbf{x}$ is the time and $\mathbf{f}$ is the height in metres $\mathbf{m}$. What is the maximum height she reaches?

Question #2

Reference Q.45521

Farmer Bob wants to build a rectangular pen for his wild sheep. Fortunately, there is a river nearby, so he can build it next to the river and avoid having to fence that one side. (His sheep are terrified of water. For some reason.) He only has 100 ft. of fencing, however. What **dimensions** should Farmer Bob use to maximize the area of the pen?

Question #3

Reference Q.45524

Your sister's drama group is putting on their annual play and you're hoping to do some marketing for them! Last year they sold 120 tickets at $10 each, but this year they want to raise the price a little. Your research shows that for every $5 dollar increase in the ticket price from last year's price, they will have 15 less people attend. Which ticket price should you recommend so that they make the most revenue?

Question #4: Applications of Quadratic Functions - Graphical

Reference Q.13772

Researchers predict that the world population will peak sometime during the $\mathbf{21^{st}}$ century before starting to decline. In the year 2000, the world population was approximately 6 100 000 000 (or 6.1 billion).

The following model has been suggested as an approximate relationship between the number of years, $\boldsymbol{x}$, since the year 2000 and the world population, $\boldsymbol{y}$.

The equation of the relationship is

$$y = -595000x^2 + 83000000x + 6100000000$$

The world population is expected to peak in the year ____.

Question #5: Perimeter of a Rectangular Land

Reference Q.13776

The perimeter of a rectangular plot of land is 84 metres and its area is 320 metres^2. If the length of the plot is represented by $\boldsymbol{x}$ metres, form a quadratic equation in $\boldsymbol{x}$, and solve it to find the length and width of the plot.

Question #6: Applications of Quadratic Functions - Graphical

Reference Q.13769

Recall the following information from Class Ex. #2 on page 348.

The hockey club hand 7 200 season ticket holders, who each paid $1 400 for a package of season tickets. The owner had suggested raising the price to generate more revenue, but knew that the number of season ticket holders would be reduced.

The general manager suggested that more revenue might be obtained by decreasing the price and thus attracting more fans to buy a package of season tickets. The research company, that the owner hired to explore the general manager's suggestion, reported that for every $50 decrease in price, approximately 400 new season ticket holders would be generated.

a. If the price decrease is to be a multiple of $50, determine the following:
   ii. the price of a package of season tickets which would generate maximum revenue
   iii. the number of season ticket holders which would be generated
   iv. the revenue which would be generated if the plan in a) was implemented

b. What advice would you give the owner in regards to the direction he should take to obtain maximum revenue?

Question #7: Applications of Quadratic Functions - Graphical

Reference Q.13771

Luigi owns a potato farm in southern Alberta. Each year he faces a dilemma as to when to harvest his crop of potatoes. He know that if he harvests early, the price will be high but his yield will be low and if he harvests late, the price will be low but the yield will be high.
From past experience, he knows that if he harvests on July 15, he can expect approximately 2000 kg of potatoes which he could sell at $0.60 per kg.
For each week he waits after July 15, he can expect an extra 400 kg of potatoes, but the price will reduce by $0.05 per kg.

When should he harvest his crop for maximum revenue?

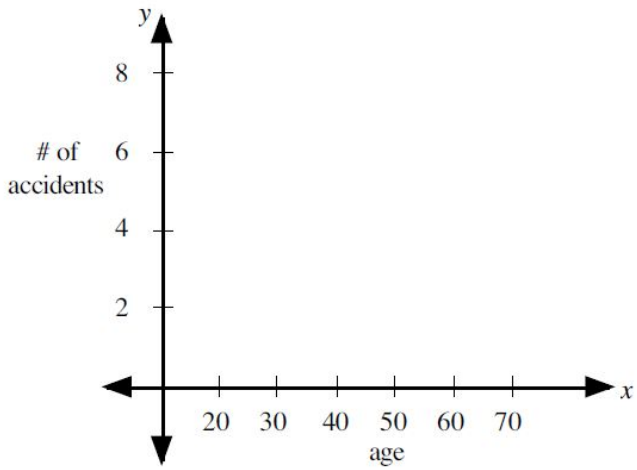
Question #8: Applications of Quadratic Functions - Graphical

Reference Q.13770

The cost of car insurance depends on many factors, one of which is the age of the driver. Insurance companies know that younger drivers under the age of 25 and older drivers over the age of 70 are statistically more likely to have accidents than drivers between the ages of 25 and 70. The following data shows the number of accidents of per million kilometres driven by drivers of a particular age.

Age (x)	18	30	45	60	75
Number of Accidents (y)	5.2	3.1	2.2	2.8	4.7

a. If x represents the age of drivers and y represents the number of accidents per million kilometres driven, plot the data on a Cartesian plane and join the points with a smooth curve.



b. The data looks like it could be modelled by a quadratic function with equation $y = ax^2 + bx + c$. Using the technique of quadratic regression (which is taught in a higher level math course), a teacher determines that the equation which best models the data is $y = 0.0034x^2 - 0.3232x + 9.8505$
Use the above model to determine what age, to the nearest year, results in the lowest number of accidents per million kilometres?

c. Determine the lowest number of accidents per million kilometres. Answer to the nearest tenth.

d. Based on this model, who is more likely to have an accident - a 17 year old student or a 78 year old senior?

Question #9: Applications of Quadratic Functions - Algebraic

Reference Q.13775

The sum of a number, x, and its reciprocal, is $\frac{29}{10}$. Form an equation and find the original number.

Question #10: Applications of Quadratic Functions - Algebraic

Reference Q.13779

Two numbers have difference of 20. When the squares of the numbers are added together, the result is a minimum. The larger of the two numbers is

A. 0
B. 10
C. 20
D. 30

Question #11: Applications of Quadratic Functions - Algebraic

Reference Q.13781

One positive integer is 3 greater than 4 times another positive integer.
If the product of the two integers is 76, then the sum of the two integers is _____.

Question #12: Applications of Quadratic Functions - Algebraic

Reference Q.13782

A whole number is multiplied by 5 and added to 3 times its reciprocal to give a sum of 16.
The number is _____.

Question #13: Applications of Quadratic Functions - Algebraic

Reference Q.13785

Use the vertex formula to determine the coordinates of the vertex of the graph of each of the following functions. State the maximum or minimum value of each function.

a. $f(x) = 5x^2 + 3x - 2$

b. $f(x) = -3x^2 - 7x - 1$

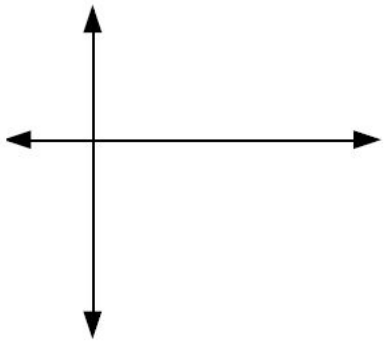
c. $f(x) = x^2 + 9x + 4$

Question #14: Applications of Quadratic Functions - Graphical

Reference Q.13768

The cross section of a river, from one bank to the other, can be represented by the function $d(w) = \frac{1}{14}w^2 - \frac{5}{7}w$ where $d(w)$ is the depth, in metres, of the river w metres from the left edge of the river bank.

a. Sketch the graph of the cross section of the river using a graphing



calculator.

b. Determine the depth of the river 3 metres from the left edge.

c. What is the maximum depth of the river, to the nearest hundredth of a metre?

d. How far from the left edge of the river, to the nearest tenth of a metre, is the deepest part of the river?

e. What is the width of the river to the nearest tenth of a metre?